3.11 Thinking Mathematically
3 hours; 3 credits

Problem solving and applications of mathematical thinking in the real world and in the ideal world of mathematics. Elementary number theory and public key cryptography. Integers, rational numbers, real numbers and the sizes of various infinite sets. Additional topic chosen from: geometry, elementary topology, chaos and fractals, probability. (Not open to students who are enrolled in or have completed Core Studies 5 or 5.2 or Mathematics 2.5 or any mathematics course numbered 3 or higher.)

Common Goals Addressed by the Core Course:

To develop the ability to think critically and creatively, to reason logically, and to reason quantitatively. \((includes OARM Goals 10 and 15)\)

To acquire the tools that are required to understand and respect the natural universe. \((includes OARM Goal 5)\)

To understand what knowledge is and how it is acquired by the use of differing methods in different disciplines. \((includes OARM Goal 17)\)

To establish a foundation for life-long learning and the potential for leadership. \((includes OARM Goals 27 and 30)\)

Objectives of Core Course

1. Students will approach mathematical puzzles or problems with a more appropriate repertoire of strategies \((OARM goal 10)\);

2. Students will understand prime factorization, Euclid’s proof of the infinitude of primes, modular arithmetic, and perform modular calculations efficiently \((OARM goals 10 and 15)\);

3. Students will understand, at least in outline, the idea of public key cryptography, and the RSA algorithm \((OARM goal 15)\);

4. Students will understand the difference between rational and irrational numbers and have some feel for how these interweave on the number line \((OARM goal 5 - knowledge goal)\);

5. Students will discuss what it means for one infinite set to be larger than another infinite set, and understand Cantor’s proof that the reals are not countable. \((OARM goal 5 - knowledge goal)\);
Outcomes for this Core Course

1. Students solve non-traditional problems in class and for homework, and discuss the strategies that led them to their solution.

2. Students compute prime factorizations and compute high powers and inverses in modular arithmetic.

3. Students participate in classroom discussions of public key cryptography. Students encode and decode messages in simple examples.

4. Students solve problems on existence of rationals or irrationals in given intervals.

5. Students solve problems on showing given sets are countable, resp. uncountable.

Assessment Methods:

Examinations and homework assignments both requiring long answers, showing all the steps with written explanations. Also, class discussion will help gauge level of class’ understanding of concepts.

Results of class tests and finals will be analyzed statistically to identify which of the outcomes for this course are being met, and to what degree. The detailed syllabus may be adjusted in light of this assessment.

Method of Evaluation: There will be two or three hour examinations, required homework assignments and a final examination. Class participation will also be considered in computing a student’s grade.

“Short answer questions” will not be used – students will be required to show, and instructors to read, all the steps in their solutions. Explanations will be required as appropriate.
Course Outline

Weeks 1 and 2: “Stories “ with mathematical content and their resolutions. Counting — the pigeonhole principal and related issues

Week 3: Patterns and Fibonacci numbers

Week 4, 5 and 6: Prime numbers and clock arithmetic. Proof of the infinitude of primes. Check digits in bar codes and ISBN’s. Fermat’s little theorem and public key (RSA) cryptography

Weeks 7 and 8 : rational and real numbers. Irrationality of 2 and related matters (this depends also on prime factorization from Weeks 4 and 5). Finding rationals and irrationals in any interval.

Weeks 8 and 9 : Why N and Q are the same size. Cantor’s diagonal argument applied to show that N is smaller than R. Cantor’s diagonal argument revisited to show that S is smaller than P(S) for any set S.

Week 10: “Exploring the infinite within geometrical objects.”

Weeks 11 through 14: Optional topic - choose one additional chapter from 4, 5, 6, 7. (which are geometric gems, topology, chaos and fractals, probability) and present some of it in depth. The Heart of Mathematics software can help reinforce any of these last four chapters.

Possible Text: The Heart of Mathematics: An invitation to effective thinking, Edward Burger and Michael Starbird, Key College Publishing in cooperation with Springer-Verlag, 2d edition, 2005

Bibliography


