**CISC 1002 [CORC 3310, CC 30.10] Paradoxes and the Limits of Knowledge**

3 hours; 3 credits

Paradoxes and limitations arising in computer science, the physical sciences, and mathematics. Paradoxes created by using reason alone. Linguistic and philosophical paradoxes like “This sentence is false.” Limitations of reason, logic and computers. Reasoning about infinity. The inability to prove everything that is true. Problems that cannot be solved by computers in a reasonable amount of time. Unsolvable problems. The boundary between what can and cannot be known. [Not open to students who are enrolled in or have completed CORC 3310]

**Goals Addressed by Core Course:**

1. Be able to think critically and creatively, to reason logically, to reason quantitatively, and to express their thoughts orally and in writing with clarity and precision.

5. Acquire the tools that are required to understand and respect the natural universe.

6. Understand what knowledge is and how it is acquired by the use of differing methods in different disciplines.

**Objectives of this Core Course:**

1. Students will be able to recognize and create self-referential paradoxes.

2. Students will be able to understand the difficulties of certain famous philosophical problems.

3. Students will be able to determine whether a set is finite, countably infinite or uncountably infinite.

4. Students will be able to tell if certain computer problems are solvable or unsolvable.

5. Students will be able to tell if a solvable problem is effectively solvable or not.

6. Students will be able to describe Gödel’s incompleteness theorem.

**Outcomes:**

1. Students will understand many self-referential paradoxes.

2. Students will know the power and the limitations of many parts of modern science.

3. Students will learn what are the effective and absolute limits of computers.

4. Students will be able to describe their work both orally and in writing.

**Syllabus:**

Week 1) Linguistic paradoxes

Basic linguistic paradoxes such as the liar paradox ("This sentence is false"). Other possible paradoxes that can be used are Russell's, Grelling's Berry's and Richard's. This will show the frailties of language and meaning. (Suggested readings from Poundstone).

Week 2) Philosophical paradoxes

Simple paradoxes about space, time, change, and movement (Zeno's paradoxes, time travel, heap paradox). (Suggested readings from Sainsbury, Salon, Wright).

Week 3-5) Infinity

Sets and their sizes will be discussed. We will show basic counterintuitive notions such as the set of all whole numbers is the same size as the set of all even numbers. Different levels of infinity. Independence of the certain properties of set theory (proven by Paul Cohen of Brooklyn College). (Suggested readings from Rucker, Ross and Wright).

Week 6) Scientific limits

Limitations of scientific knowledge such as the butterfly effect. The implications of Bell's theorem, Heisenberg's uncertainty principle, the collapse of the wave function. (Suggested readings from GellMann, Mermin, Morton).

Week 7-9) Computers and their effective limits

Easy and hard problems that computers can solve. Certain problems that can be solved by computers but not in any reasonable amount of time (such as the traveling salesman problem, the Hamiltonian graph problem). The relationship between these problems. (Suggested readings from Davis).

Week 10-11) Computers and their absolute limits.

Problems that a computer can never solve. Halting problem, printing problem. (Suggested readings from Davis).

Week 12-14) Mathematical limitations

Mathematical problems that are unsolvable. Trisecting an angle, solving the quintic, Gödel's incompleteness theorems. (Suggested readings from Barrow, Dawson, Hofstadter, Nagel and Newman).

**Bibliography:**


- Barrow, John D., *Impossibility: The Limits of Science and the Science of Limits*, Oxford University Press, USA (October 28, 1999).
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- Carroll, Lewis, "What the Tortoise Said to Achilles," *Mind* 4, No. 14, Pg. 278-280, April 1895. Available at http://www.ditext.com/carroll/tortoise.html
- Martin Davis, "What is a computation", in *Mathematics Today*, Lynn Arthur Steen, Vintage Books (Random House), 1980.
- Davis, Martin and Hersh, Reuben, "Hilbert's 10th Problem," *Scientific American*, November, 1971.
- Davis, Phillip J. and Hersh, Reuben, *The Mathematical Experience*, Mariner Books, 1999.
- Dawson, John W., "Gödel and the limits of logic", *Scientific American*, June 1999.
- Feferman, Solomon, "Deciding the Undecidable: Wrestling with Hilbert's problems", in *In the Light of Logic*.
- Gell-Mann, Murray, *The Quark and the Jaguar : Adventures in the Simple and the Complex,* Holt Paperbacks; Illustrate edition, 1995.
- Hofstadter, Douglas R., *Gödel, Escher, Bach: An Eternal Golden Braid*. Basic Books; 20 Anv edition (February 4, 1999).
- Kline, Morris, *Mathematics: The Loss of Certainty*, Oxford University Press, 1982.
- Lewis, David, "The Paradoxes of Time Travel", in van Inwagen and Zimmerman's *Metaphysics: The Big Questions*.
- Mermin, N. David, "Is the moon there when nobody looks? Reality and the quantum theory," Physics Today, April 1985, pp.38-47. [Also available at: http://xoomer.virgilio.it/baldazzi69/papers/mermin−moon.pdf]
- Morton, Tavel, *Contemporary Physics and the Limits of Knowledge*, Rutgers University Press, July, 2002.

- Nagel, Ernest and Newman, James R., "Gödel's Proof", *Scientific American*, June, 1956.
- Poundstone, William, Labyrinths of Reason: Paradox, Puzzles, and the Frailty of Knowledge, Anchor, December, 1989.
- Ross, Kenneth A., Wright, Charles R.B., section 13.3 "Infinite Sets" of *Discrete Mathematics*, Fifth Edition. Prentice Hall, 2003.
- Rucker, Rudy, Infinity and the Mind: The Science and Philosophy of the Infinite, Princeton Science Library, 2004.
- Smullyan, Raymond M., *To Mock a Mockingbird*, Oxford University Press, 2000.
- Sainsbury, R.M., *Paradoxes 2nd ed.,* Cambridge University Press, 1995.
- Salon, Wesley C.,"A Contemporary Look at Zeno's Paradoxes: an Excerpt from *Space, Time and Motion*" in van Inwagen and Zimmerman's *Metaphysics: The Big Questions*.
- Wright, Crispin. "Language-mastery and the sorites paradox." In *Truth and Meaning: Essays in Semantics*. Edited by Gareth Evans and John Henry McDowell. Oxford, UK: Clarendon Press, 1976.


**Text:**
The text of the course will be a course-pack that will consist of several of the articles in the references and chapters from books. The articles will be on the level of Scientific American articles.

**Web Resources:**
- http://ocw.mit.edu/OcwWeb/Linguistics-and-Philosophy/24-118Fall-2006/Readings/index.htm

- http://www.nd.edu/~jspeaks/courses/20229/syllabus.html

**Method of Assessment:**
1. Students will be asked to answer short answer questions as well as multiple choice questions. There will also be short essay questions.

2. Students will be presented with infinite sets and will be asked if these sets are finite, countably infinite or uncountably infinite.

3. Students will be asked to produce their own self-referential paradoxes.

4. Students will be presented with computer problems and will be asked to determine if they are effectively solvable or not.

5. Students will have to write a short paper on some contemporary discussion about a limitation of computation or modern science.

**Methods of evaluation:**
The two tests (25% x 2), a short paper (15%) and a cumulative final (35%) will be used to assign grades.