Name:

PART I: Answer all 6 questions. Each question is worth 10 points. Justify each answer and show all your work.

1. (a) Find all \( x \in \mathbb{R} \) satisfying the equation \( -x = \sqrt{3 + 2x} \) and check the validity of your answer.
   (b) Write \( \frac{2+3i}{3-5i} \) in the form \( a + bi \), where \( a \) and \( b \) are real numbers.

2. (a) Find an equation of the straight line that passes through the point \((-1, 2)\) and perpendicular to the line with equation \(2x + 4y = -5\).
   (b) Find the center and radius of the circle with equation \(x^2 + y^2 - 4x + 2y + 1 = 0\)
   and sketch its graph showing its center and at least four points on it.

3. (a) Find the maximal domain of \( g(x) = \sqrt{x^2 - 3x} \) for all real values. Write the solution using interval notation and sketch it on the real number line.
   (b) Start with the graph of \( y = e^x \) and use transformations to sketch the graph of \( y = -e^{x+1} - e \). Label all intercepts.

4. (a) Find all zeros (roots), real or complex, of the polynomial \( P(x) = x^3 - x^2 + 9x - 9 \) and give the complete factorization of \( P(x) \).
   (b) Sketch the graph of \( y = f(x) = x^2 + 4x - 5 \).
   Identify its vertex and \( x \) and \( y \) intercepts, and label each of these coordinate points. Find the domain and range of \( f(x) \)? At what interval on the domain is \( f(x) \) increasing? decreasing?

5. (a) Simplify and express \( \log_3(81) + 27^{-\frac{3}{4}} \) as a rational number.
   (b) Find all \( x \in \mathbb{R} \) satisfying \( \frac{1}{256} = 4^{3x} \).

6. (a) Given \( f(x) = \frac{1}{x-1} \) and \( g(x) = \frac{1}{x} \). Find \( (g \circ f)(x) \) and state its domain. Find \( (f \circ g)(\frac{1}{3}) \).
   (b) Find the inverse, \( f^{-1}(x) \), of the function \( f(x) = \frac{2}{x+3} \).

Please turn over!
PART II: Answer 4 out of 6 questions. Each question is worth 10 points. Justify each answer and show all your work.

7. 

Shown is a parabola with vertex at \((2, -1)\). It has \(y\) intercept at \((0, 3)\).
(a) Find an equation of the form \(y = ax^2 + bx + c\) for this parabola.
(b) Where does this parabola meet the \(x\) axis?

8. (a) Find the center, vertices, and the length of the minor axis of the ellipse \(4(x + 3)^2 + 25(y - 1)^2 = 100\) and sketch its graph.
(b) Find the term involving \(y^2\) in the binomial expansion of \((2x + y)^5\).

9. (a) Verify the identity \(\cos^4(x) - \sin^4(x) = \cos(2x)\).
(b) Without using a calculator, simplify and express \(\tan[\arccos(\frac{\sqrt{3}}{2})]\) in terms of rational numbers or radicals.

10. (a) Find the amplitude and period of \(y = 5 \cos(3x)\), and graph one complete period.
(b) Let \(f(x) = -2x^2 + 5x - 4\). If \(h \neq 0\), the expression
\[
\frac{f(x+h) - f(x)}{h}
\]
can be re-expressed as a polynomial in \(x\) and \(h\). Give such a polynomial.

11. (a) Find an equation for the circle taking the line segment between two points \(P(-6, -4)\) and \(Q(-2, -8)\) as a diameter.
(b) Find \(\cos(A + B)\) if \(A\) is in the second quadrant with \(\sin(A) = \frac{4}{5}\) and \(B\) is in the fourth quadrant with \(\cos(B) = \frac{12}{13}\).

Please turn over!
12. (a) For the function given by $f(x) = \frac{2x-9}{x+3}$:

(i) Find the $x$ and $y$ intercepts of the graph of $f(x)$.

(ii) On what interval is $f(x) = \frac{2x-9}{x+3} > 0$? $f(x) = \frac{2x-9}{x+3} < 0$?

(iii) Find the vertical and horizontal asymptotes of $f(x)$.

(iv) Use the information above to sketch the graph of $f(x)$.

(b) Find all $x \in \mathbb{R}$ satisfying $\log_4(2x - 3) = \log_4(x) + \log_4(x - 2)$ and check the validity of your answer.

End of Examination