

Brooklyn College  
The City University of New York  
Department of Mathematics

CC 3.11, Au 2008  
Thinking Mathematically

Tuesdays, Thursdays, 11:00-12:15  
3 credits

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Office hours: by appointment

*“The College-wide Core Curriculum, a mandatory requirement for all candidates for a baccalaureate degree, is intended to provide students with a broad background in the liberal arts and sciences. Students select and complete, during the four years of their college career, a mandatory set of eleven interrelated courses, plus a foreign language requirement, from a specially developed, two-tier grouping of core courses. These are foundation courses designed for non-specialists and suitable for non-majors, but each is planned so as to introduce material of fundamental and lasting significance.*

*The Core Curriculum aims to broaden awareness, cultivate the intellect, and stimulate the imagination, rather than to provide specific career preparation. The courses are intended to develop mental skills, rather than vocational skills, and in this respect, the courses constitute the best possible long-term preparation for any career.”*

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**Bulletin description:**

Problem solving and applications of mathematical thinking in the real world and in the ideal world of mathematics. Elementary number theory and public key cryptography. Integers, rational numbers, real numbers and the sizes of various infinite sets. Additional topic chosen from: geometry, elementary topology, chaos and fractals, probability. (Not open to students who are enrolled in or have completed Core Studies 5 or 5.2 or Mathematics 2.5 or any mathematics course numbered 3 or higher.)

**Common Goals Addressed by the Core Course:**

- To develop the ability to think critically and creatively, to reason logically, and to reason quantitatively. (*includes OARM Goals 10 and 15*)
- To acquire the tools that are required to understand and respect the natural universe. (*includes OARM Goal 5*)
- To understand what knowledge is and how it is acquired by the use of differing methods in different disciplines. (*includes OARM Goal 17*)
- To establish a foundation for life- long learning and the potential for leadership. (*includes OARM Goals 27 and 30*)

**Objectives of Core Course**

- Students will approach mathematical puzzles or problems with a more appropriate repertoire of strategies (*OARM goal 10*);
- Students will understand prime factorization, Euclid’s proof of the infinitude of primes, modular arithmetic, and perform modular calculations efficiently (*OARM goals 10 and 15*);
- Students will understand, at least in outline, the idea of public key cryptography, and the RSA algorithm (*OARM goal 15*);
- Students will understand the difference between rational and irrational numbers and have some feel for how these interweave on the number line (*OARM goal 5 – knowledge goal*);
- Students will discuss what it means for one infinite set to be larger than another infinite set, and understand Cantor’s proof that the reals are not countable. (*OARM goal 5 - knowledge goal*);

### Outcomes for this Core Course

- Students solve non-traditional problems in class and for homework, and discuss the strategies that led them to their solution.
- Students compute prime factorizations and compute high powers and inverses in modular arithmetic.
- Students participate in classroom discussions of public key cryptography. Students encode and decode messages in simple examples.
- Students solve problems on existence of rationals or irrationals in given intervals.
- Students solve problems on showing given sets are countable, uncountable.

### Attendance:

Prompt and regular attendance is a requirement of this course. If you are absent from class five times, you will not be eligible to pass the course.

As soon as you either know you are going to be absent, or have been absent, you should e-mail me to, at the very least, acknowledge that you should have been in class and to find out what you should do to make up for the absence.

Most in class assignments (pop quizzes, etc.) are given at the very start of class; you will not be given additional time or allowed to make these up if you show up late.

### Plagiarism:

CUNY Policy on Academic Integrity

“Academic Dishonesty is prohibited in the City University of New York and is punishable by penalties, including failing grades, suspension, and expulsion, as provided herein.” -- CUNY Policy on Academic Integrity, Adopted by the Board of Trustees 6/28/2004

Please go to <http://www.brooklyn.cuny.edu/bc/policies/> for further information about:

- CUNY Policy on Academic Integrity
- BC Procedures for Implementing the CUNY Policy on Academic Integrity
- Flow Chart of the BC Procedures for Implementing the CUNY Policy on Academic Integrity.

### Assessment:

Two Midterms (1 hour each)	25%
One Final Examination	25%
Pop quizzes and papers	15%
Weekly homework	15%
Attendance	20%

## Course Outline

### *Weeks 1 and 2:*

“Stories “ with mathematical content and their resolutions. Counting — the pigeonhole principal and related issues

### *Week 3:*

Patterns and Fibonacci numbers

### *Week 4, 5 and 6:*

Prime numbers and clock arithmetic. Proof of the infinitude of primes. Check digits in bar codes and ISBN’s. Fermat’s little theorem and public key (RSA) cryptography

### *Weeks 7 and 8:*

rational and real numbers. Irrationality of  $\sqrt{2}$  and related matters (this depends also on prime factorization from Weeks 4 and 5). Finding rationals and irrationals in any interval.

### *Weeks 8 and 9:*

Why  $\mathbb{N}$  and  $\mathbb{Q}$  are the same size. Cantor’s diagonal argument applied to show that  $\mathbb{N}$  is smaller than  $\mathbb{R}$ . Cantor’s diagonal argument revisited to show that  $S$  is smaller than  $P(S)$  for any set  $S$ .

### *Week 10:*

“Exploring the infinite within geometrical objects.”

### *Weeks 11 through 14:*

Optional topic - choose one additional chapter from 4, 5, 6, 7. (which are geometric gems, topology, chaos and fractals, probability) and present some of it in depth. *The Heart of Mathematics* software can help reinforce any of these last four chapters.

## Assigned Text:

*The Heart of Mathematics: An invitation to effective thinking*, Edward Burger and Michael Starbird, Key College Publishing in cooperation with Springer-Verlag, 2d edition, 2005

## Bibliography

Blitzer, R. *Thinking Mathematically*, Prentice Hall, 2d edition, 2003

Cannon, Heal, Wellman, and Duffin, CD-ROM: *The Heart of Mathematics* interactive explorations, 2000-2004

Johnson, D, and Mowry, T. *Mathematics: A Practical Odyssey*, Brooks Cole, 2001, 4th edition

Miller, C, Heeren, V., and Hornsby, J., *Mathematical Ideas*, Pearson, 2004, 10th edition