Math 2001 - A Transition to Advanced Mathematics

3 hours; 3 credits

Course objectives: At the time of completing the course, students are expected to

1. obtain the basic skills to understand and write mathematical proofs;
2. understand the elements of mathematical language;
3. understand the logic in mathematical proofs;
4. understand rigorously the completeness of the real number system and its equivalence with the least upper bound property;
5. understand how the well-ordering property of the natural numbers implies the principle for mathematical induction; and finally
6. obtain the skills to learn advanced mathematics courses such as advance calculus, abstract algebra, geometry and topology.

Prerequisite: Math 1206

Textbooks:

I (required). *Mathematical Proofs: a Transition to Advanced Mathematics*, by G. Chartrand, A. D. Polimeni and P. Zhang, Addison Wesley Publication.

II (recommended). *An Introduction to Abstract Mathematics*, by R. J. Bond & W. J. Keane, Brooks/Cole Publication.

Course Outline

1. How to communicate in mathematics, and sets — 2 weeks (I. Chaps. 0 & 1)
   a. Structure and guidelines of mathematical writing
   b. Common words and phrases in mathematics
   c. Sets, subsets and set operations
   d. Indexed collections, partitions and Cartesian products of sets
2. Logic — 2 weeks (I. Chap. 2)
   a. Statements, negations, disjunctions and conjunctions of statements
   b. Implications, and biconditional
   c. Tautologies and contradictions, logical equivalence
   d. Characteriations of statements, quantified statements and their negations
3. Direct proof and proof by contradiction — 1 week (I. Chap. 3, secs. 1-4)
   a. Trivial and vacuous proofs, direct proofs
   b. Proof by contrapositive, proof by cases
4. Proofs in elementary number theory — 1 week (I. Secs. 11.1-2 & secs. 4.1-2)
   a. Divisibility properties of integers and related proofs
   b. The division algorithm and proofs involving congruence of integers
5. Proof by contradiction and disprove — 1 week (I. Secs. 5.1-2 & secs. 6.5-6)
   a. Proof by contradiction
   b. Counterexamples and disproving statements
6. Relations — 1 week (I. Chap. 7)
   a. Relations, reflective, sysmmetric, transitive, and equivalent relations
   b. Properties of equivalent classes and examples
7. Functions — 1 week (I. Chap. 8)
   a. Definition, the collection of functions, one-to-one, onto, and bijective functions
   b. Composition of functions and inverse functions, permutations
8. Mathematical induction — 1 week (I. Secs. 9.1-6)
   a. The well-ordering principle and induction principle, and related examples of proofs
   b. More examples of proofs
9. Cardinalities of sets, and the real number system — 2 weeks (II. Secs. 6.1-2, secs 7.1-2)
   a. Countable set, uncountable sets, and the Schroeder-Bernstein Theorem
   b. Development of the real number system, and its uncountability
   c. The order and completeness of the real number system, and Cauchy sequences
   d. The least upper bound property of the real number system and related