SYLLABUS FOR

MATHEMATICS 18.1 - ABSTRACT ALGEBRA II

I. Groups.
- Subgroups. Subgroup generated by a subset. Cyclic groups and their subgroups.
- Symmetric groups.
- Homomorphism of groups.
- Order of an element in a group.
- Coset of decomposition of a group. Index of a subgroup.
- Lagrange theorem on order of a subgroup.
- Normal subgroup.
- Kernel of a homomorphism. Relation between homomorphisms and normal subgroups. Quotient groups.

II. Rings
- Homomorphisms and Isomorphisms of rings.
- Subrings; embedding a ring in a ring with identity.
- Ideals. Residue class ring modulo an ideal. Quotient rings.
- Relation to homomorphism.
- Basis of an ideal. Principal ideal. Prime and maximal ideals.
- Integral domains and fields.
- Quotient field of an integral domain.
- Characteristic of a ring.
- Characteristic of an integral domain is 0 or a prime. Prime subfield of a field.

III. Factorization in Euclidean domains (more generally, in principal ideal domains).
- Units, associates, primes. Highest common factor. Euclidean algorithm.
- Uniqueness of factorization.

IV. Polynomials over integral domains and fields.
Properties.
Factor and remainder theorems. Division algorithm.

As much of the following as time permits.

V. Field extensions.

- Degree of an extension. If F ? E ? K, then [k: E]. [E: F]
- Adjunction to a field.
- Degree of an element over a field. Minimal polynomial. If a is algebraic of degree n, then F (a) is algebraic over F of degree n. If $a_1, a_2, \ldots\ldots, a_n$ are algebraic over F, so is F $(a_1, a_2, \ldots\ldots, a_n)$.
- Applications to ruler and compass constructions.

VI. Introduction to Galois theory.

- Root fields. The Galois groups.
- Separable and inseparable polynomials.
- Properties of the Galois group. Fundamental theorem of Galois theory. Applications to solutions of equations.