MATHEMATICS 46

OBJECTIVES OF THE COURSE

This is **NOT** a course about high-school mathematics, Sequential 1, 2 and 3, or the practicalities of classroom teaching.

In this course I want to show you some approaches to the understanding and teaching of mathematics which I hope you will be able to use--

--by having, in my own teaching of you, an example which you can choose to follow or diverge from;

--by using your experience as a student to give you insight into how to help the students whom you will teach.

You should be aware that ideas about

--curriculum,

--the structure of classroom space and time,

--methods of assessing children's progress

are changing rapidly in official circles; some of these ideas will be tried in this course. You may be expected to learn, and then teach, in ways quite different from those you are used to.

From time to time in this course you may find yourself feeling--

--anxious
--confused
--bored
--totally lost
--inadequate
--despairing
--angry at me
--angry at yourself

--curious
--enthusiastic
--proud of something you've accomplished

One objective of this course is to give you opportunities to--

--recognise these feelings in yourself;

--discuss them openly and constructively;

--prepare to put what you learn about yourself to the best use when you become a teacher.

By the way, I encourage you to keep a **maths journal**, in which you express your feelings about this course, me, and your relationship to the course. It's up to you how often you write in it; and of course you may keep it **absolutely private**, unless *you* choose to show (portions of) it to me or any of your fellow-students. The main reasons for keeping a maths journal are:--

--many students of maths have found that keeping a journal helps them cope with their negative feelings, and enables them to continue their struggle toward mastery and self-confidence;

--you may find it very instructive at the end of the semester, or later in life, to see how your attitudes towards maths and yourself changed during this semester. More immediately, you may be able to use your journal in writing your term paper.

THE STRUCTURE OF MATHEMATICS

I think that there is a rough hierarchy in levels of understanding of mathematics--or any other subject--and corresponding modes of learning.

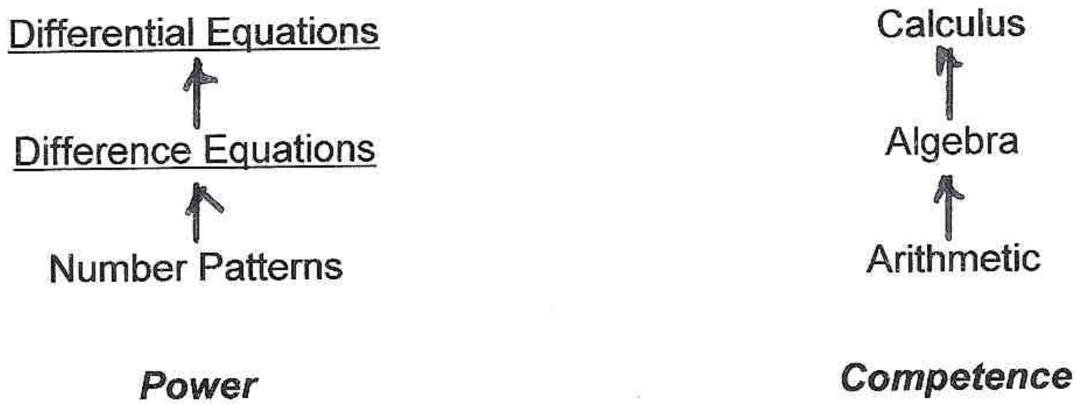
	English	Mathematics	Learning Mode
Power	Prayer, poetry, legal brief, political speech	Unity of all maths, aesthetic beauty, power of the quantitative world view	Inspiration from a teacher (or book)
Pleasure	Reading for fun, creative writing and speaking	Experiments in science and probability based on maths, maths-based games, puzzles	Cooperative learning, discovery by students
Competence	Understanding a lease or instruction manual, writing a resume, correct spelling and grammar	Arithmetic and algebraic skills, ability to convert verbal situations into maths problems	Constant practice/ drill

I hope that in this course you will experience all three levels of learning/understanding maths, and of going back and forth between them.

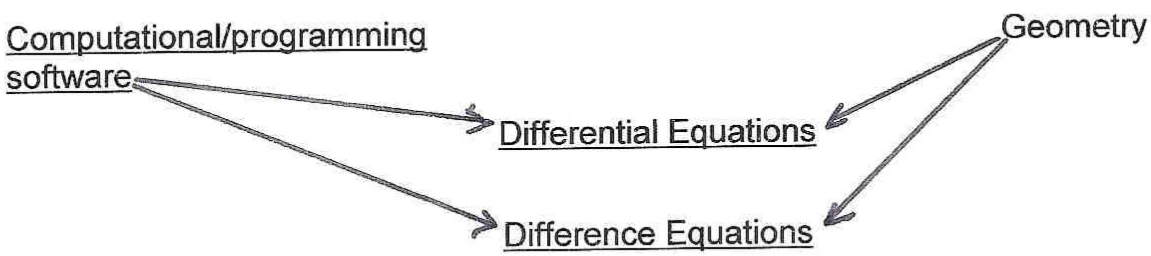
--Drill is necessary to become competent enough to enjoy the pleasurable part of maths and to appreciate its power.

--Having an overview of maths gives you guidance in selecting drill and experiments/games so as best to direct children towards future goals.

Another dimension to the structure of mathematics is by level of knowledge--by grade level, roughly speaking. For example, in the chart below (topics underlined will be studied in this course), arithmetic leads to algebra, which leads to calculus; the corresponding progression in the area of applications goes from number patterns to difference equations and then to differential equations.



It would take **another dimension or two** to express the interconnections between various areas of mathematics. For example, in many cases the same real-world situation can be studied as a differential equation or as a difference equation; and both geometry and computational software such as Mathematica can be used to understand differential equations and difference equations.



I hope in this course to give you a sampling of all these dimensions of mathematics.

MATHEMATICAL CONTENT OF THE COURSE

From the perspective(s) outlined above, there are certain areas of maths that are absolutely crucial--

--to applications in science and understanding the world
--to interconnections with other branches of maths.

Here is a list of some of these topics; again, the underlined ones are those we shall study (a little bit) in this course:

--<u>Differential equations</u> and partial differential equations
--The <u>algebra</u>, <u>geometry</u> and calculus of the complex number system
--<u>Linear algebra</u> and its applications, such as to linear differential equations and to <u>linear programming</u>

--Group theory, especially the study of symmetries in geometry
--Probability
--Certain areas of discrete maths, such as graph theory
--Topology
--Computational/programming software, such as Mathematica, Maple, Matlab, Geometer's Sketchpad.

Two other topics, difference equations and fractals, while perhaps less important, fit so well into the structure of the course that I don't want to leave them out.

PERFORMANCE OBJECTIVES OF THE COURSE

By the end of the course, students will perform successfully in the following areas:

1. **Induction**
(a) Demonstrate their understanding of the strategy of Induction by producing clearly-written proofs using that method;
(b) Formulate conjectures and prove them using Induction.

2. **Algebra and Geometry of Complex Numbers**
(a) Perform arithmetic operations on complex numbers, both algebraically and geometrically, and, in either case, both by hand and using Mathematica;
(b) Solve problems involving de Moivre's Theorem and its applications to trigonometric functions;
(c) Solve problems using the Fundamental Theorem of Algebra, in particular, solve quadratic equations with complex coefficients, both by hand and using Mathematica.

3. **Differential Equations**
(a) Solve simple differential equations by hand, both using analysis and by plotting slope fields;
(b) Plot vector fields and solve more difficult differential equations using Mathematica;
(c) Convert a real-world problem to a differential equation, solve it and interpret the solution in terms of the original problem.

4. **Difference Equations**
(a) Solve first- and second-order difference equations by hand, both numerically and algebraically;
(b) Use Mathematica to solve difference equations;
(c) Convert a real-world problem to a difference equation, solve it and interpret the solution in terms of the original problem.

5. **Linear Programming**
(a) Solve linear programming problems with two variables by hand geometrically, and problems in two or three variables by hand using Gaussian elimination;
(b) Use Mathematica to solve linear programming problems;

(c) Convert a real-world problem to a linear programming problem, solve it and interpret the solution in terms of the original problem.

6. **Newton's Method and Fractals**
(a) Use Newton's method to find approximate solutions of an equation, both analytically and geometrically;
(b) Implement Newton's method using Mathematica;
(c) Use Mathematica to construct the fractal Newton's basins of a simple polynomial;
(d) Construct self-similar fractals and compute their perimeters and areas (if defined).