Mathematics Gateway Interventions
Spring 2014  Brooklyn College
Math 1011 (Pre-calculus) Recitation
Week 9 Problem set # 9

1) (a) Discuss
   (i) the Remainder and Factor Theorem
   (ii) the Rational Zeros (Roots) Theorem
   (iii) the Fundamental Theorem of Algebra
   (iv) the Complete Factorization Theorem

(b) What is meant by
   (i) "Synthetic Division"
   (ii) "the vertical asymptote of a graph of a rational function f"
   (iii) "the horizontal asymptote of a graph of a rational function f"
   (iv) "an exponential function f" and the "natural exponential function f"
   (v) "a logarithmic function f" and the "natural logarithmic function f"
   (vi) "solutions to two system of equations with two unknowns x and y"

2) (a) Find all zeros (roots) real or complex of the polynomial \( f(x) = x^3 + 2x^2 + 4x + 8 \)
   and give the complete factorization of \( f(x) \).

(b) For the function given by \( f(x) = \frac{2x}{x-1} \):
   (i) Find the x and y intercepts of the graph of \( y = f(x) \).
   (ii) Find the vertical and horizontal asymptotes of the graph of \( f(x) \).
   (iii) Use the information above to sketch the graph of \( f(x) \).

(c) Solve for \( x \): \( \left( \frac{1}{3} \right)^{x+1} = 27 \).

(d) Solve for \( x \) and check: \( \log_e x^2 - \log_e 9 = 0 \).

(e) Solve the system of equations:
   \[-4x + 3y = 23\]
   \[12x + 5y = 1\]

3) Suppose you were asked to solve the following two problems on a test.
   (i) Find the remainder when \( 6x^{1000} - 17x^{562} + 12x + 26 \) is divided by \( x + 1 \)
   (ii) Is \( x + 1 \) a factor of \( x^{562} - 3x^{400} + x^9 + 2 \)?

   Obviously, it's seems "impractical" to try to solve these problems by dividing by hand because the polynomials are of such large degree. Use one or more of the theorems that you are familiar with to solve these problems without actually dividing.