Brooklyn College, Department of Mathematics

Final Examination, Spring 2008, Mathematics 2.9 Precalculus

Name of Instructor_____ **Name**_____

Show all work and justify your answers, so that partial credit may be given. Good luck and do well!

PART I. Do **all** problems in this part. (50 points)

1. (a) Give an equation for the perpendicular bisector of the segment between the two points $P(-5,-2)$ and $Q(7,-6)$.

   (b) A circle has segment $\overline{PQ}$ above as diameter. Give an equation for this circle.

2. Consider the polynomial $P(x) = 2x^3 - 13x^2 + 26x - 10$.

   (a) The rational root theorem produces a list of rational numbers containing all (if any) rational roots of the equation $2x^3 - 13x^2 + 26x - 10 = 0$. Give this list.

   (b) Given that a positive fraction (non-integer) is a root of the equation $2x^3 - 13x^2 + 26x - 10 = 0$, find all roots (real or complex) of the equation.

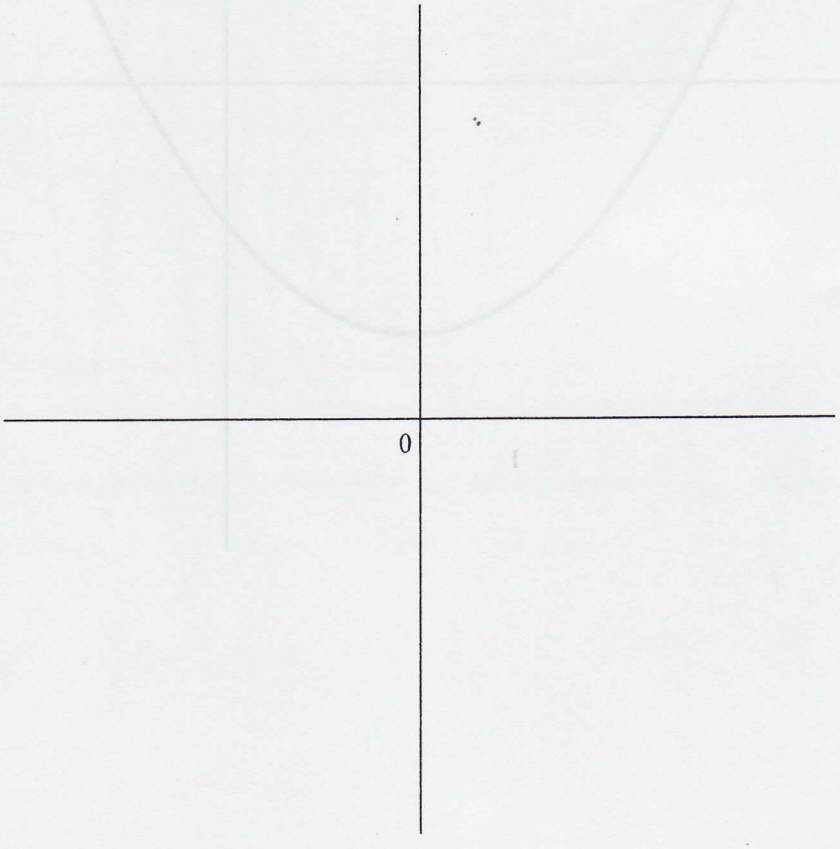
3. You are given the function $f(x) = \dfrac{3x-4}{2x-6}$.

(a) Where are the x intercepts of the graph of this function? Where are the y intercepts?

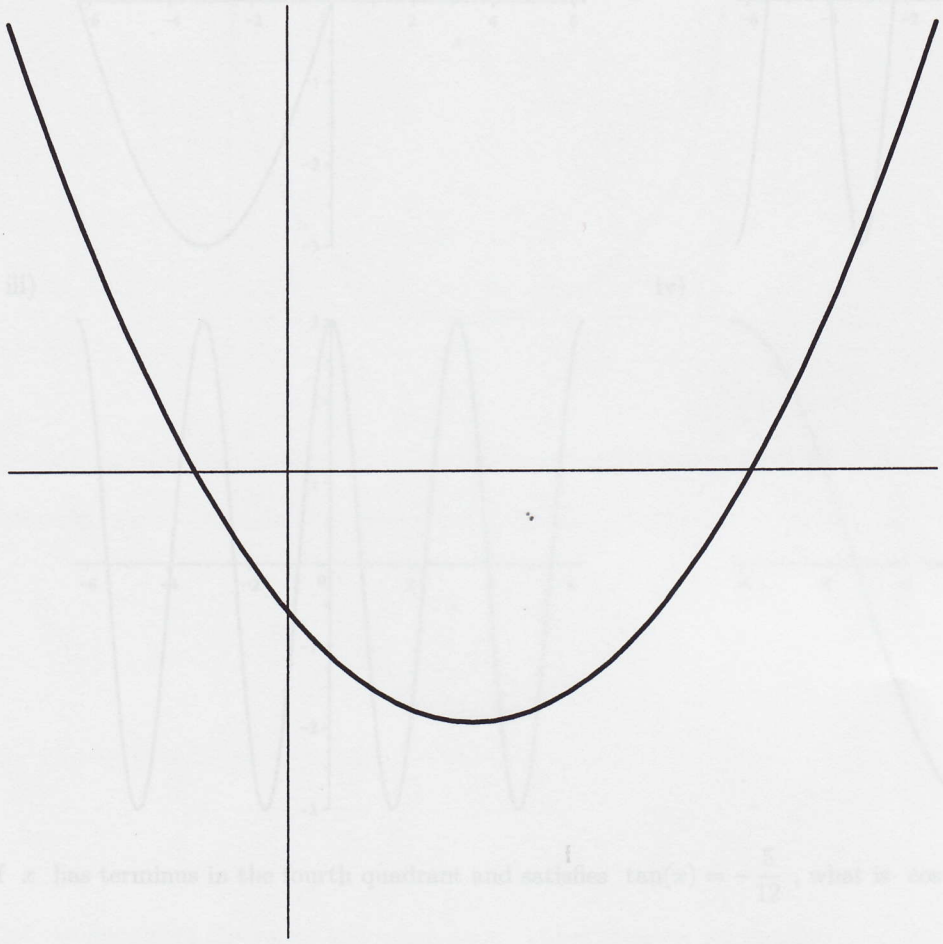
(b) For which values of x is $f(x) > 0$. Express your answer in interval notation.

(c) Give equations of all vertical asymptotes of the graph. Give equations of all horizontal asymptotes of the graph.

(d) Sketch the graph of this function in the space provided, labelling the intercepts and asymptotes.



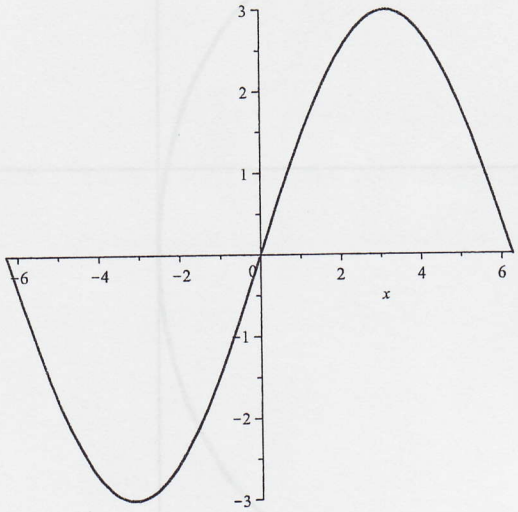
4. A parabola has equation $y = 3x^2 - 12x - 15$. This curve is shown below. Determine the coordinates of the vertex, and x and y intercepts of the parabola and label these points on the curve.



5. (a) What is the period of the function $f(x) = -3\cos(2x)$? What is the amplitude?

(b) Which of the following four curves has equation $y = f(x)$ for $x \in [-2\pi, 2\pi]$?

i)



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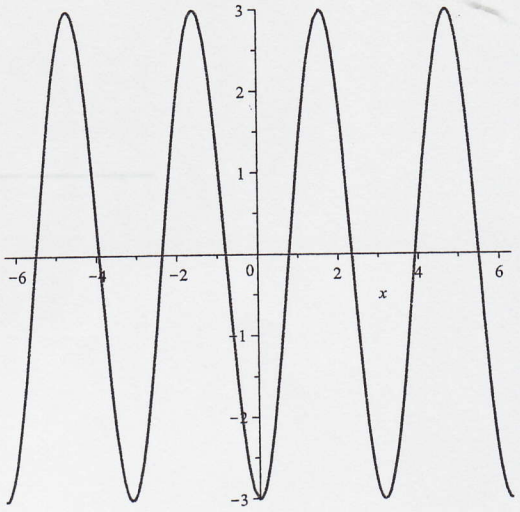
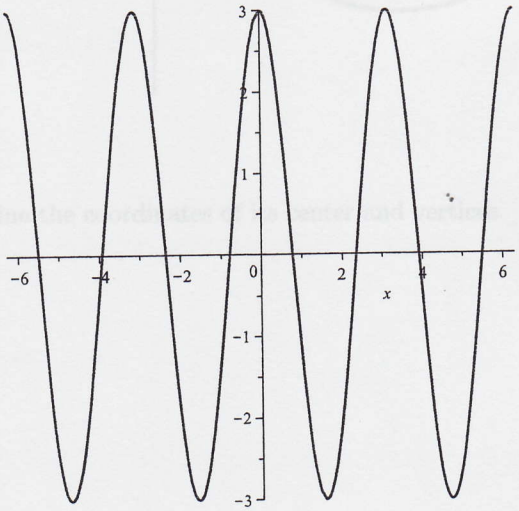
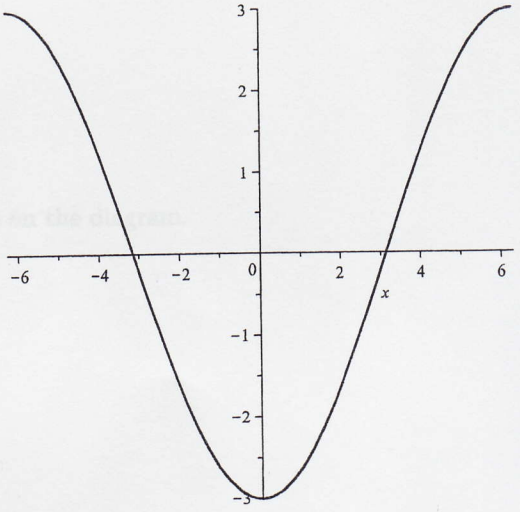


iii)



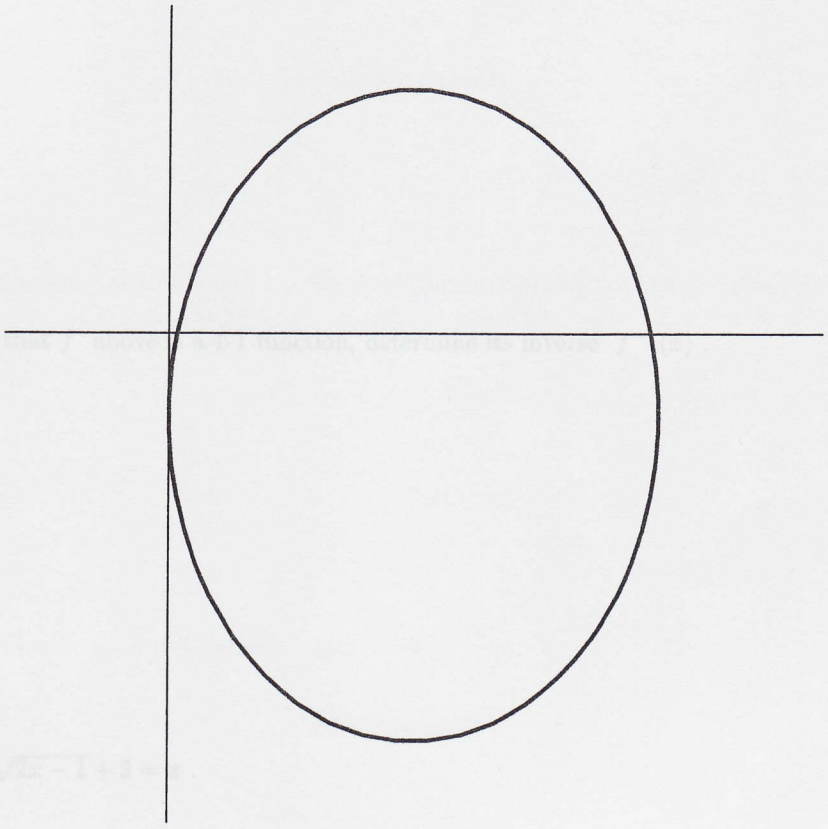
iv)



(c) If x has terminus in the fourth quadrant and satisfies $\tan(x) = -\frac{5}{12}$, what is $\cos(2x)$?

PART II. Do **any five** of the following six problems. (50 points)

6. Shown is the ellipse with equation $\dfrac{(x-3)^2}{9} + \dfrac{(y+1)^2}{16} = 1$.



(a) Determine the coordinates of its center and vertices. Label these on the diagram.

(b) Find the lengths of the major and minor axes. Label these on the diagram.

7. Given the functions $f(x) = \dfrac{2x-3}{x+2}$ and $g(x) = 2x^2 + x - 3$,

(a) Evaluate $g \circ f(-3)$. What is $f \circ g(x)$?

(b) Given that f above is a 1-1 function, determine its inverse $f^{-1}(x)$.

8. (a) Solve $\sqrt{2x-1} + 2 = x$.

(b) Solve $32^{x+1} = 8^x$.

9. (a) Solve $\log_4(x+5) + \log_4(x-1) = 2$.

(b) Evaluate $\tan\left(\arccos\left(-\frac{4}{9}\right)\right)$. Decimal approximations are not acceptable.

10. (a) Prove that $(1+\sin(\theta)) \cdot (\sec(\theta) - \tan(\theta)) = \dfrac{\cot(\theta)}{\csc(\theta)}$.

(b) Evaluate $\csc\left(\frac{5\pi}{4}\right)$. Decimal approximations are not acceptable.

11. (a) What is the domain of the function $f(x) = \dfrac{\sqrt{2x-1}}{x^2-4}$?

(b) With $g(x) = 3x^2 - 8$, simplify $\dfrac{g(x+h) - g(x)}{h}$ (assuming $h \neq 0$).

(c) Express $\dfrac{3+4i}{2-3i}$ in the form $a + b \cdot i$, where a and b are real numbers.

PRINT NAME. LAST: _____ FIRST: _____

DEPARTMENT OF MATHEMATICS
BROOKLYN COLLEGE

FINAL EXAMINATION—SPRING 2007
MATHEMATICS 2.9

PART I. *Do* **all** *problems in this part. (52 points).*

(10 pts) 1. (a) Find an equation of the line going through the point $(5, 3)$ and perpendicular to the line $2x - 8y + 4 = 0$.

(b) Find the equation of a circle that goes through the point $(1, 5)$ and has the point $(-2, 1)$ as its center.

(11 pts) 2. (a) Find the vertex and all x-intercepts and all y-intercepts of the graph of the function $f(x) = -3x^2 - 12x + 15$.

(b) Graph $f(x)$.

(10 pts) 3. (a) Find all solutions of the equation $\log_2(4-x)+\log_2(x+5)=3$.

(b) Find all solutions of the equation $1-\sqrt{3x+3}=2x$.

(10 pts) 4. (a) Verify the identity $\dfrac{1}{1+\cos\theta} + \dfrac{1}{1-\cos\theta} = 2\csc^2\theta$.

(b) Find $\cot\left(\arccos\left(-\dfrac{12}{13}\right)\right)$.

(11 pts) 5. (a) Find the coordinates of the center and of the vertices of the ellipse $\dfrac{(x+4)^2}{16} + \dfrac{(y-2)^2}{25} = 1$.

(b) Graph the ellipse.

PART II. *Do any* **four** *of the five problems in this part. (48 points).*

(12 pts) 6. (a) Find the inverse of the function $f(x) = \dfrac{4x+2}{3x-1}$.

(b) Find the domain and range of f.

(c) Let $g(x) = 3x^2 - 2x + 4$ and $h(x) = x + 2$. Find $g \circ h(x)$.

(12 pts) 7. (a) Writing $P(x) = 3x^3 - 4x^2 - 2x + 1$, list all possible rational solutions allowed by the Rational Roots Test of the equation $P(x) = 0$.

(b) Find all solutions the equation $P(x) = 0$.

(c) Write the polynomial $P(x)$ as a product of two (non-constant) polynomials *with integer coefficients.*

(12 pts) 10. (a) Find the term involving x^8 in the binomial expansion of $(2x^4 - 3)^5$.

(b) Given the function $f(x) = \dfrac{x^2 + 3}{(x+3)(2x-1)}$, find its (i) horizontal and (ii) vertical asymptotes.