This is a collection of questions that will be representative of those on the final exam. This is **NOT** a sample final exam: the final exam may have more or fewer questions, or the questions may be asked in a different format.

There **WILL** be other types of questions on the final exam. However, about 80% of your grade on the exam will be based on questions similar to the following. (The remainder will be based on everything else you did in the course)

Circles

Question C1

A drone flies in a circular path around an object that is 100 feet east and 50 feet north of the drone operator. The drone's path takes it over a point that is 40 feet east and 30 feet south of the drone operator.

Find an equation for the drone's path.

Does the drone cross a north-south line through the operator's position? If so, where? If not, explain why.

Suppose the drone operator is facing the object. Is the drone ever "behind" the drone operator? Why/why not?

Question C2

A drone flies in a circular path around an object that is 100 feet east and 50 feet north of the drone operator; the drone is always 75 feet from the object.

Find an equation for the drone's path.

Does the drone cross a north-south line through the operator's position? If so, where? If not, explain why.

Suppose the drone operator is facing the object. Is the drone ever "behind" the drone operator? Why/why not?

Functions

Question F1

Let f be the function defined by $f(x) = \sin x + \cos x$ and let g be the function defined by $g(u) = \sin u + \cos u$, for all real numbers x and u. Which of the following are true:

a) f and g are exactly the same functions

b) if x and u are different numbers, f and g are different functions
c) not enough information is given to determine if f and g are the same.

Question F2

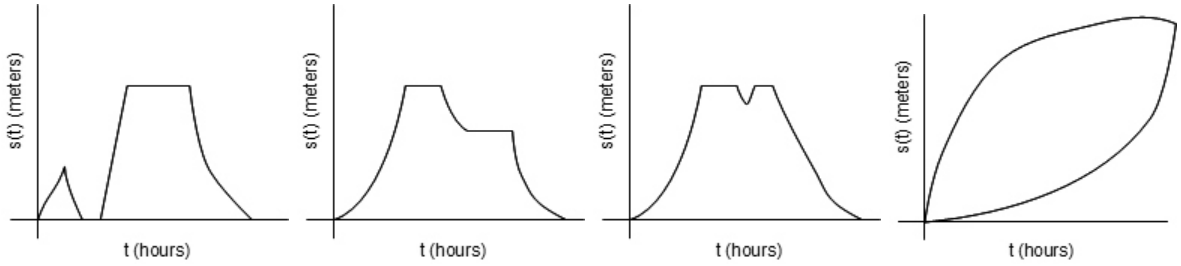
Suppose you go visit a friend. You go there, then you and your friend go out to eat. After you finish eating, you return home. Let S(t) be your distance from home in meters, *t* hours after you start your trip.

Find S(0). Defend your answer.

What is the meaning of S(5)? (Don't try to find the value; just determine what its value would mean).

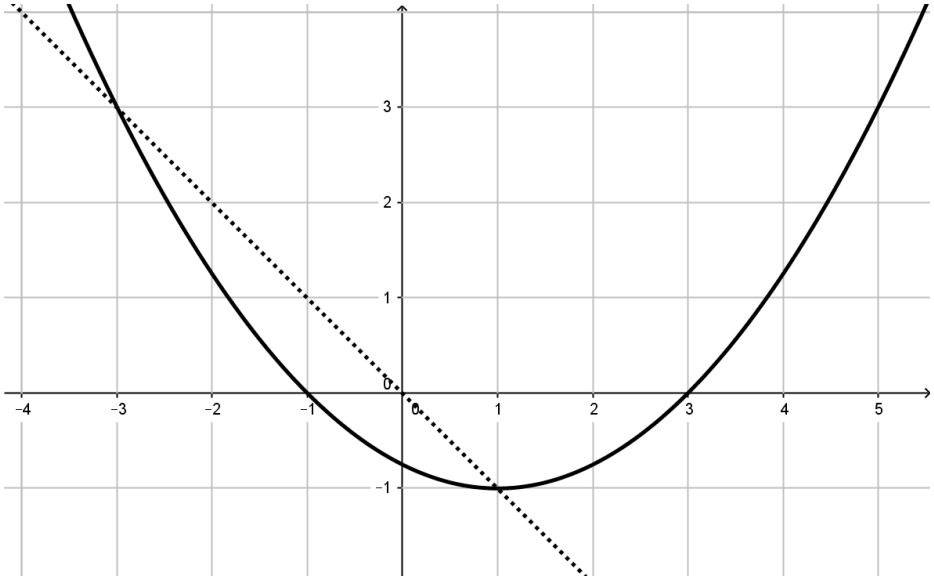
Could S(t) ever be negative? Why/why not?

Which of the following might be the graph of y = S(t)?



Question F3

The graphs of y = f(x) (solid) and y = g(x) dotted) are shown.



Find the domain and range of f(x).

Find the domain and range of $h(x) = \sqrt{f(x)}$.

Find the domain of k(x) = $\frac{1}{f(x)}$.

For what values is f(x) larger than g(x)?

For what values is f(x) increasing?

Determine: (f + g)(-3), (fg)(-3), (f - g)(-3).

Does g(x) have an inverse? If it does, graph the inverse; if it does not, explain why not.

Find the average rate of change of f(x) over the interval [-3, 1].

Question F4

The cost of a data plan is $45 a month, plus $0.40 per gigabyte of data downloaded. Let f(x) be the total cost of the data plan when you download x gigabytes in a month. To pay for your data plan, you enroll in autopay through your bank. However, your bank charges a "convenience" fee: Every payment you make costs $2, plus 3% of the payment amount.

Let g(x) be the total cost of the convenience fee for a payment of $x.

Write an algebraic expression for f(x) and g(x).

Find f(g(10)).

What, if any, is the meaning of f(g(10))?

Find g(f(10)).

What, if any, is the meaning of g(f(10))?

Find the average rate of change of the convenience fee as the number of gigabytes downloaded goes from 5 to 10 gigabytes.

Question F5

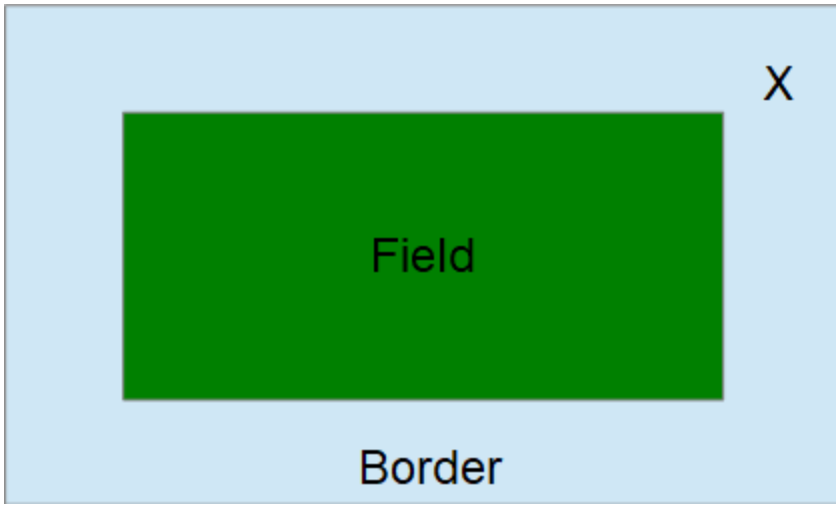
Suppose you want to put a fence around a rectangular field, where the width is five times the length.

Find an expression for the perimeter of the field in terms of its length.

Is the **perimeter** of the field function of its length? Explain why/why not.

Is the **area** of the field a function of the length? Explain why/why not.

Suppose the field has a length of 25,000 feet. You buy fencing to enclose it, but discover you've bought 10 feet too much. Rather than throwing away the extra fencing, you enclose a slightly larger field by enlarging each side of the field by the same amount, so the fencing encloses the field and a border:

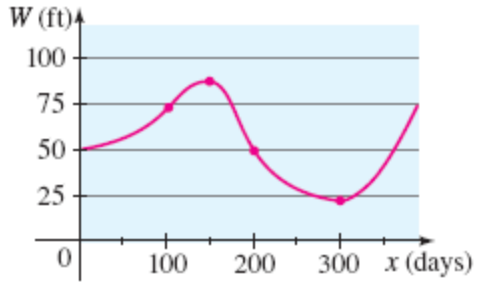


What is the largest animal that could be at point X and be entirely within the border region, touching neither the fence nor the field? (a) an ant, (b) a mouse, (c) a pigeon, (d) a cat, (e) an elephant.

Find the average rate of change of the perimeter, if the length increases from x feet to x + h feet.

Question F6

The graph shows the depth of water W in a reservoir over a one-year period as a function of the number of days x since the beginning of the year. What is the domain and the range of the function W? Where is the function increasing? Decreasing? What was the average rate of change of W between x = 0 and x= 100?



Parabolas

Question P1

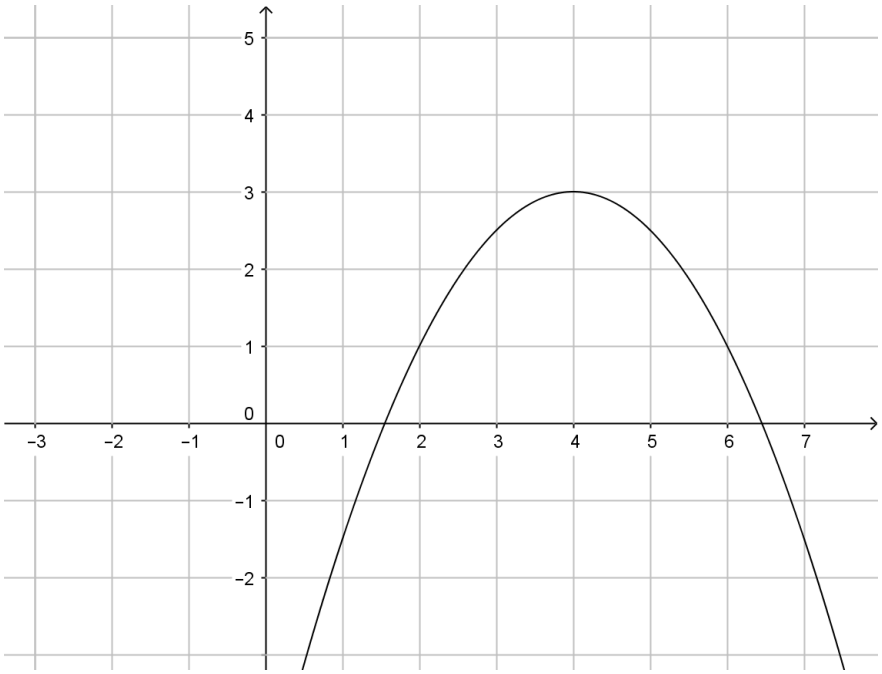
Suppose the graph of $y = ax^2 + 3x + c$ intersects the x-axis at x = 2 and x = 8. Find the location of the vertex and the y-intercept.

Question P2

A cannon fires a shell. The shell moves along a parabolic trajectory whose highest point is 1200 feet in the air. The shell lands 800 feet away from the cannon. Write a function h(t) giving the height of the shell as a function of the *horizontal* distance from the cannon.

Question P3

A parabola has equation $y = ax^2 + 3x + c$. The graph is shown below.



Find the *exact* (algebraic) values of the x- and y-intercepts.

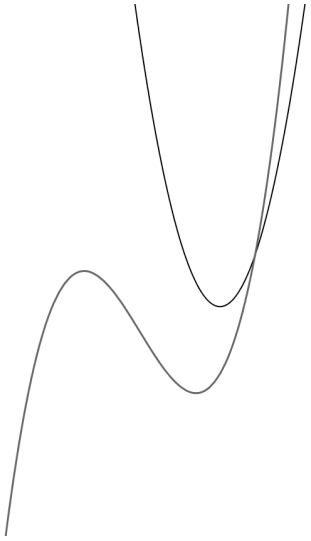
Polynomial Equations

Question E1

The graphs of $y = 5x - 3$ and $y = 2x^3 + 3x^2 + px$ intersect at x = 1. Find the remaining intersection points.

Question E2

The graph of y = f(x) and y = g(x) are shown (you may assume f, g are polynomials with real coefficients, and one is the graph of a quadratic and the other is the graph of a cubic, and that there are no additional turning points).



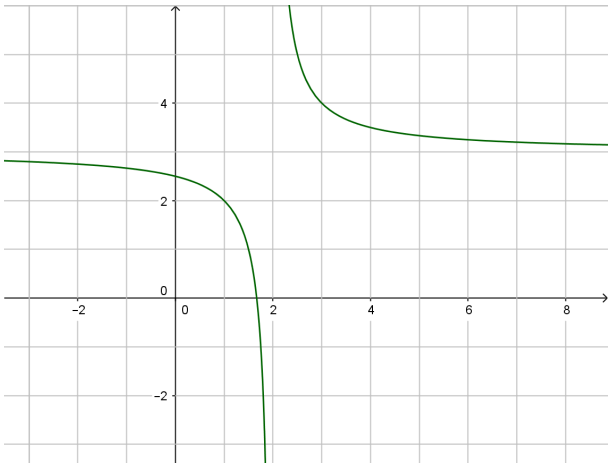
Based on the graphs, the equation f(x) = g(x), which of the following are true?

- There is one real solution and no complex solutions.
- There is one real solution and one complex solution.
- There is one real solution and two complex solutions.
- There are three real solutions.

Rational Functions and Inequalities

Question R1

The graph of y = f(x) is shown. Assume f(x) is a rational function.



Find all x where f(x) is undefined. Defend your conclusion.

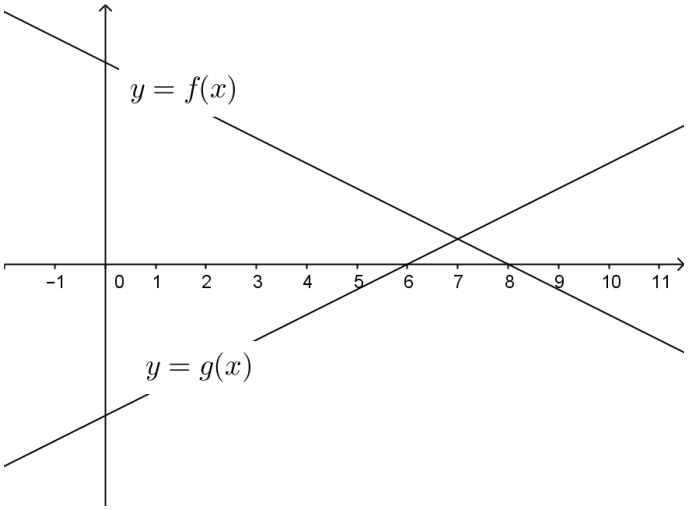
Give equations for the vertical and horizontal asymptotes, if they exist.

Describe appropriate transformations that can be used to produce the graph of y = f(x) from the graph of y = $\frac{1}{x}$. Then find an equation for f(x). (Assume that only translations and reflections are used).

Find where f(x) = 0.

Question R2

The graphs of y = f(x) and y = g(x) are shown below. **DO NOT** attempt to find an equation for either f(x) or g(x).



WITHOUT finding an equation for f(x) or g(x), answer the following:

Let $R(x) = \frac{f(x)}{g(x)}$.

Where is R(x) zero?

Where is R(x) undefined?

Find an equation for the vertical asymptote(s) of R(x).

Where is R(x) positive?

Where is R(x) negative?

Determine whether the horizontal asymptote of y = R(x) is above or below the x-axis. Defend your conclusion.

Question R3

If possible, write the equation of a rational function with all of the following properties:

- Horizontal asymptote of y = 5,
- Vertical asymptotes of x = 3 and x = -8
- Continuous for all x not equal to 3 or -8.

Question R4

True or False: If $f(x) = \frac{(x-2)(x+2)}{(x-2)}$ and g(x) = x + 2, then f(x) = g(x).

Logarithms and Exponents

Question L1

Suppose $\log_k p = 5, \log_k q = -2$. Find the following.

$\log_k(p^3q^2)$

k^8 (express your answer in terms of p and/or q)

Suppose $\log_k r$ = 9. Find r in terms of p and/or q.

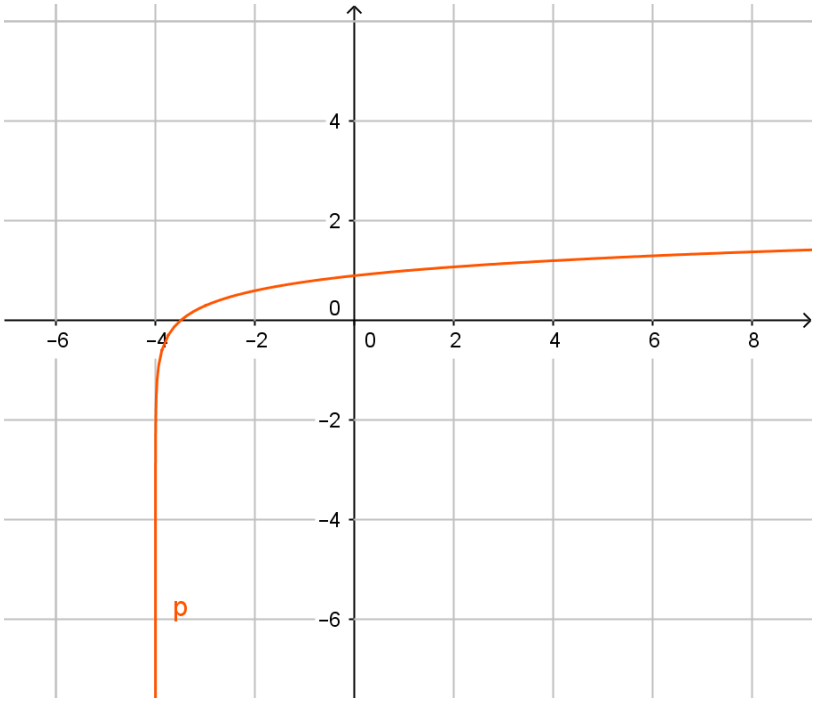
Question L2

The graph of y = f(x) is shown; the graph goes through the point (1, 1). Provide two *different* graph transformations that produce the graph of y= f(x) as follows:

First, provide a sequence of transformations of the graph of y = log(x) that produce the graph. This sequence should use **only** horizontal shifts and stretches.

Second, provide a different sequence of transformations that produce the graph. Your second sequence of transformations **must** include a vertical shift or stretch.

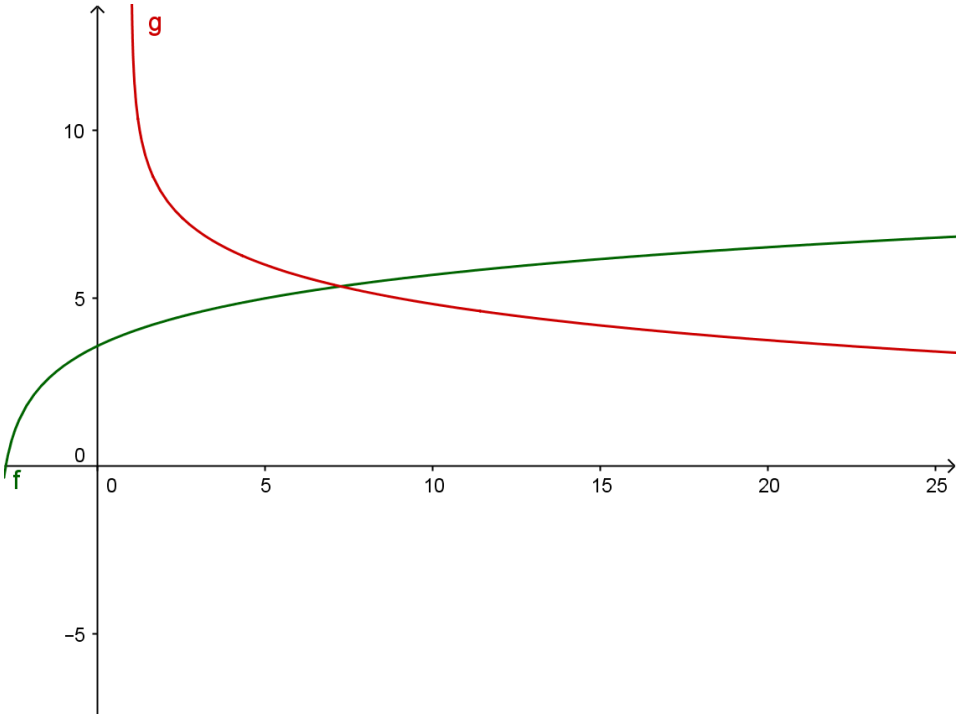
Your grade will be based on how accurately your described transformations reproduce the graph shown.



Question L3

The graph shows two transformations of the graph of $\log_2 x$:

- One is the graph of y = $\log_2 x$ shifted horizontally by 3 units to the right, then 4 units vertically.
- The other is the graph of y = $\log_2 x$ shifted 4 units to the left, flipped horizontally, then shifted 8 units vertically.



Write the equations of both graphs.

Identify which graph goes with which equation.

Determine the intersection point.

Find all horizontal and vertical asymptotes, if they exist.

Question E1

A disease spreads through a population. The number of cases t days after the start of the epidemic is shown below.

Days after start (t)	56	64
Number infected ($N(t)$ thousand)	6	12

Assume the disease spreads at an exponential rate.

How many cases will there be on day 77? ☍ thousand (Round your answer to the nearest thousand)

On approximately what day will the number infected equal ninety thousand? ☍ (Round your answer to the nearest whole number)

Trigonometry

Question T1

How long is the shadow cast by a 34-foot tree on the (horizontal) ground when the sun angle is 26 degrees?

Question T2

Let $x = 2\tan\theta, 0 \leq \theta \leq \frac{\pi}{2}$. Express in terms of the sine and/or cosine of θ.

$$\frac{x}{4+x^2}$$

$$\frac{\sqrt{4+x^2}}{x^2}$$

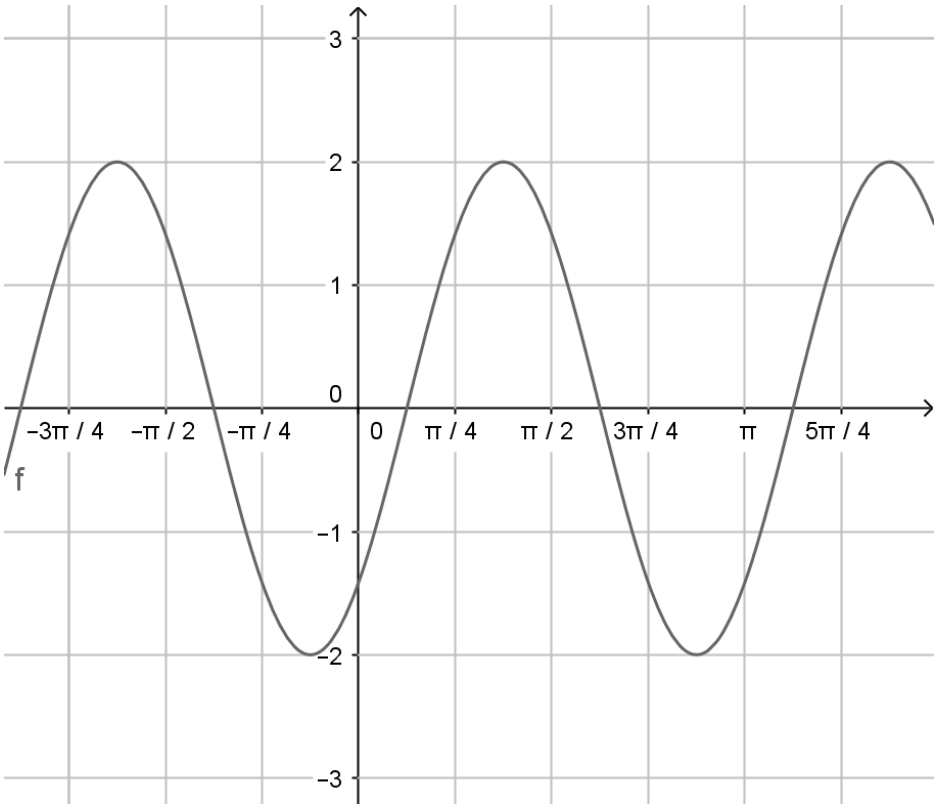
Question T3

Suppose $\sin 2x = \frac{3}{5}$. Find all possible values of tan x, cos x, ...

Suppose $\cos x = -\frac{3}{5}$. Find all possible values of sin 2x, tan 2x, cos 2x...

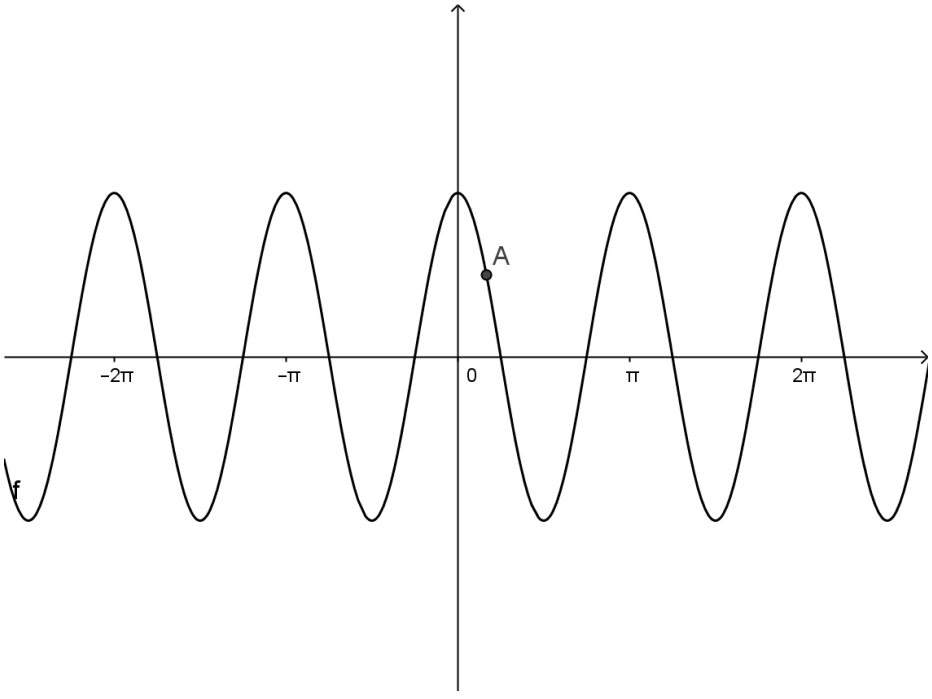
Question T4

The graph of y = f(x) is shown below. Describe how transformations can be used to produce it from the graph of y = sin x. Then determine the period, phase shift, and amplitude; find the first positive x for which f(x) = 1. Finally, describe how it can be produced by transforming the graph of y = cos x.



Question T5

The graph shows a transformation of y = sin x. The labeled point has coordinates $\left(\frac{\pi}{6}, 3\right)$.



Determine the transformations required to obtain the graph shown from the graph of y = sin(x).

Write the equation for the graph shown.

Give the coordinates of the first positive intersection point of the graph and the x-axis.

Give the coordinates of the first positive peak of the graph.

Describe how you could obtain the graph shown form the graph of y = cos(x).

Binomial Theorem

Question B1

Find the indicated terms in the expansion of

$$(4z^2 - z + 2)(10z^2 - 5z - 4)(5z^2 - 5z - 4)$$

The degree 5 term is

The degree 1 term is

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Question B2

Suppose the expansion of

$$(4x^2 + x + 5)(cx - 5)(4x^2 + 4x + 3)$$

includes the term $44x^3$ after all like terms have been combined.

Then $c =$

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Question B3

Suppose the binomial expansion of $(2x + 4y)^n$ includes the term $35840x^4y^3$.

The x^5y^2 term in the expansion will be

The x^3y^6 term in the expansion will be

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