PART I. Do all problems in this part.

(10 pts) 1. (a) Find the equation of a circle a diameter of which has endpoints \((-3, 6)\) and \((7, 2)\). Sketch the graph of the circle.

(b) Find an equation of the line going through the point \((4, -5)\) and perpendicular to the line \(2x + 8y - 9 = 0\).
(10 pts) 2. (a) Find the vertex of the parabola $y = 4x^2 - 40x + 107$ and sketch its graph.

(b) Find the domain and range of the function $y = 4x^2 - 40x + 107$. 
(11 pts) 3. (a) Prove the identity \( \frac{2\tan x}{1 + \tan^2 x} = \sin 2x \).

(b) Given that \( \cot t = 3/4 \) and that \( t \) is in the fourth quadrant, find \( \cos t \).
4. (10 pts) (a) Find the coordinates of the foci and the equations of the asymptotes of the hyperbola $4y^2 - 25x^2 + 150x - 32y - 261 = 0$.

(b) Graph the hyperbola.
(11 pts) 5. (a) Find all solutions of the equation $\log_{10}(x - 5) + \log_{10}(x + 16) = 2$.

(b) Find all solutions of the equation $4x - \sqrt{8x + 1} - 7 = 0$. 
PART II. Do any four of the five problems in this part. Each problem is worth 12 points.

6. (a) Find the inverse of the function \( f(x) = \frac{2x + 1}{x - 3} \).

(b) Find the domain and range of \( f \).

(c) Let \( g(x) = 2x^2 - x + 4 \), and \( h(x) = x + 2 \). Find \((g \circ h)(x)\).
7. (a) Find the binomial expansion of \((3x^3 - y^2)^4\).

(b) Given the function \(f(x) = \frac{3x^2 + 1}{(x + 3)(x - 2)}\), find its (i) horizontal and (ii) vertical asymptotes.
8. (a) Find the equation of the ellipse with vertices \((6, 4), (3, 9),\) and \((3, -1)\).

(b) Find the foci of the ellipse, and graph the ellipse.
9. (a) What is the period of the function \( f(x) = 3 \cos 2x \)? Use increments of \( \pi/6 \) or smaller on the \( x \)-axis to sketch the graph of \( f(x) \) in the interval \([0, \pi]\).

(b) Find (i) \( \arctan(-\sqrt{3}) \), and (ii) \( \arccos(-\sqrt{3}/2) \).

(c) Find all solutions between 0 and \( 2\pi \) of the equation \( \cos t = -1/2 \).
10. (a) Given $P(x) = 4x^3 - 12x + 5x + 6$, list all possible rational solutions allowed by the Rational Roots Test of the equation $P(x) = 0$.

(b) Find all solutions the equation $P(x) = 0$. (*Hint: $x = 2$ is a solution of this equation.*)

(c) Write the polynomial $P(x)$ as a product of linear factors with integer coefficients.