PART I: Answer all 6 questions. Each question is worth 10 points. Show your work and justify your answer for each problem.

1) (a) Find an equation of the line that passes through the point \((-5, -2)\) and is perpendicular to the line \(-5x - 2y = 27\). Write the equation in slope–intercept form.

(b) Find the domain of the function \(f(x) = \sqrt{x - 1} + 5\). Then, identify the transformations made from \(g(x) = \sqrt{x}\) to \(f(x) = \sqrt{x - 1} + 5\).

2) (a) Write the quadratic function \(f(x) = -x^2 + 2x + 8\) in the standard form. Identify the vertex, and find the \(x\)-intercepts and \(y\)-intercept of the graph. Then, sketch the graph of the function. Show the points for the vertex and intercepts clearly on the graph.

(b) Solve the following exponential equation algebraically:

\[
\left(\frac{1}{2}\right)^{2(1-3x)} = \frac{1}{8}
\]

3) Consider the rational function \(f(x) = \frac{-x + 4}{x - 5}\):

(i) Find the vertical and horizontal asymptotes, if any exist
(ii) Find the \(x\)-intercept and \(y\)-intercept
(iii) Using the information obtained from (i) and (ii) above, graph the function

4) (a) Let \(f(x) = x^2 - 3x - 5\). Find the difference quotient of the function \(f(x)\), that is, find the expression for:

\[
\frac{f(x + h) - f(x)}{h}
\]

and simplify the result.

(b) Without using a calculator, simplify and express \(y = \sin (\arctan 2)\) in terms of rational numbers or radicals:

5) (a) Find the inverse, \(f^{-1}(x)\), of the function \(f(x) = \frac{-6x + 8}{8x - 3}\). Then, find the domain of the inverse function \(f^{-1}(x)\).

(b) Divide the complex numbers. Write the result in \(a + bi\) form.

\[
\frac{4 + i}{-3 - 5i}
\]
6) Consider the two functions \( f(x) = \frac{1}{x - 3} \) and \( g(x) = \sqrt{x} \):

(i) Find the composite function \( (f \circ g)(x) \)
(ii) Find the domain of \( (f \circ g)(x) \)
(iii) Evaluate \( (f \circ g)(4) \)

Part II: Answer 4 out of 5 questions. Each question is worth 10 points. Show your work and justify your answer for each problem.

7) (a) Verify the trigonometric identity:
\[
\frac{\sin x}{1 - \cos x} + \frac{\sin x}{1 + \cos x} = 2 \csc x
\]

(b) Solve the inequality \( \left| \frac{3 - 2x}{5} \right| < 7 \) and write the solution set in interval notation.

8) (a) Solve the logarithmic equation \( \log (4 + x) - \log (x - 4) = \log 5 \). After you find the possible solution(s), make sure to do the check.

(b) Write a polynomial function of minimum degree with real coefficients whose zeros are: 4, 1 + 3i, and 1 - 3i.

9) (a) Find the center, vertices, and foci of the ellipse
\[
\frac{(x + 4)^2}{36} + \frac{(y - 3)^2}{100} = 1
\]
then sketch its graph.

(b) Find the amplitude, period, and phase shift of \( y = -3 \sin \left( \frac{1}{2}x - \pi \right) \). Sketch the graph in one complete period.

10) (a) Find the center and radius of the circle with the equation
\[3x^2 + 3y^2 + 6y - 12x - 2 = 0\]
then sketch its graph, clearly indicating its center and the 4 endpoints.

(b) Find all solutions of the trigonometric equation \( \sin^2 x + \sin x = 0 \) in the interval \([0, 2\pi]\).

11) (a) Expand the binomial \( (3x - \frac{1}{2})^5 \) and write the term involving \( x^3 \) and also the term involving \( x^1 \).

(b) If \( \cos \theta = \frac{2}{3} \) and \( \tan \theta < 0 \), without using a calculator, simplify and express the value of \( \sin \theta \) in terms of rational numbers or radicals.