MATHEMATICS, BROOKLYN COLLEGE
FINAL EXAMINATION
MATH 1011, FALL 2016

NAME: _____

Solve **ten** of the following eleven problems. Show all work for full credit. No calculators, cellphones, or other electronic devices permitted. (Each problem worth 10 points, total: 100 points.)

1. You are given a segment with endpoints $P(-5,-1)$ and $Q(7,5)$.

   (a) Find an equation for the perpendicular bisector of this segment.

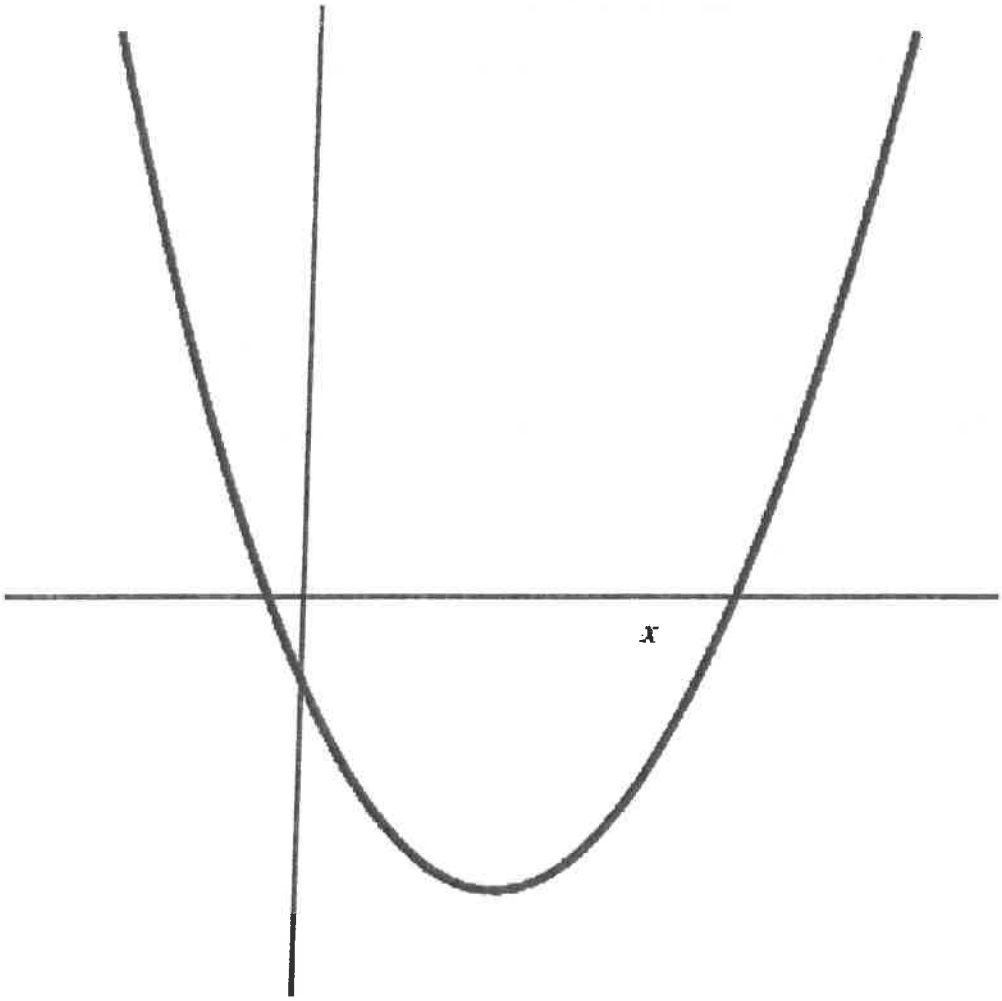
   (b) The segment is a diameter of a circle. Determine the center and radius of this circle.

   (c) Find an equation of form $x^2 + y^2 + A\,x + B\,y + C = 0$ for this circle determined in part (b).

2. Consider the parabola with equation $y = 2x^2 - 8x - 3$.

   (a) Write an equation of the form $y = b(x - h)^2 + k$ for the parabola.

   (b) The plot below shows the parabola. Identify (label with coordinates) its vertex, its x-intercepts, its y-intercept, and (label with equation) its axis of symmetry.



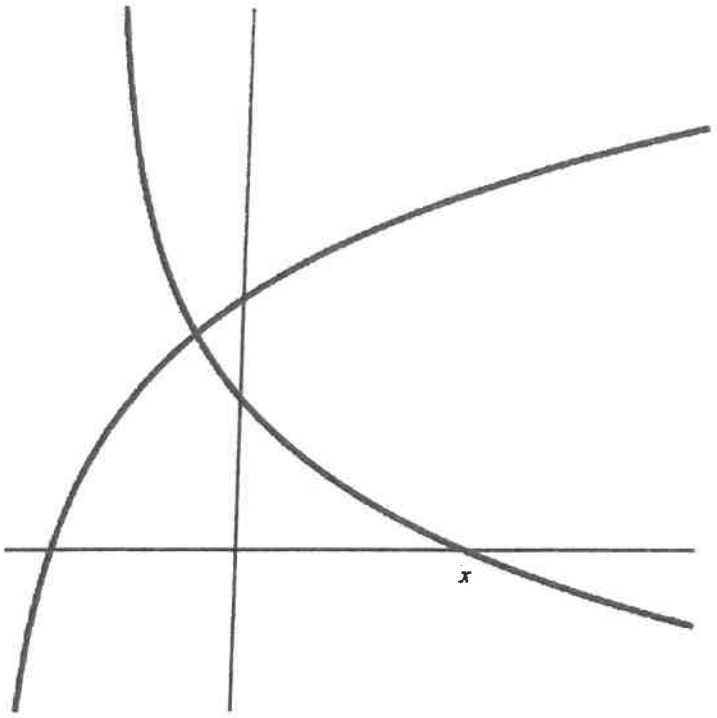
3. You are given functions $f(x) = \sqrt{x+5}$ and $g(x) = \frac{1}{x}$.

(a) Evaluate $(f \circ g)(1)$ and $(g \circ f)(4)$.

(b) What is the domain of f? What is the domain of g?

(c) Determine the function $(g \circ f)(x)$ and give its domain.

4. Shown are the graphs of $f(x) = \log_2(x+5)$ and $g(x) = 3 - \log_2(x+3)$.



(a) On the diagram, match the graphs to the functions.

(b) Identify (by labelling with coordinates the appropriate points) the x intercepts of the two curves.

(c) Identify (by labelling with coordinates) the intersection of the two curves.

5. Angle φ is in standard position with terminus in the second quadrant. You are given that $\sin(\varphi) = \frac{1}{3}$.

(a) Determine the value of $\cos\left(\arcsin\left(\frac{1}{3}\right)\right)$ in terms of rational numbers and (possibly) radicals.

(b) Evaluate $\tan(\varphi)$.

(c) Express $\cos(2\varphi)$ in terms of rational numbers and (possibly) radicals.

(d) Verify that for any admissible angle θ the following is true:

$$\frac{1}{\sin^2(\theta)} - 2 = \frac{1 - \tan^2(\theta)}{\tan^2(\theta)}$$

6. The polynomial equation $x^3 + x - 10 = 0$ has at least one rational root.

(a) Give the list of potential rational roots supplied by the rational root test.

(b) Find the three roots of the equation.

(c) What is the product of the three roots determined in part (b)?

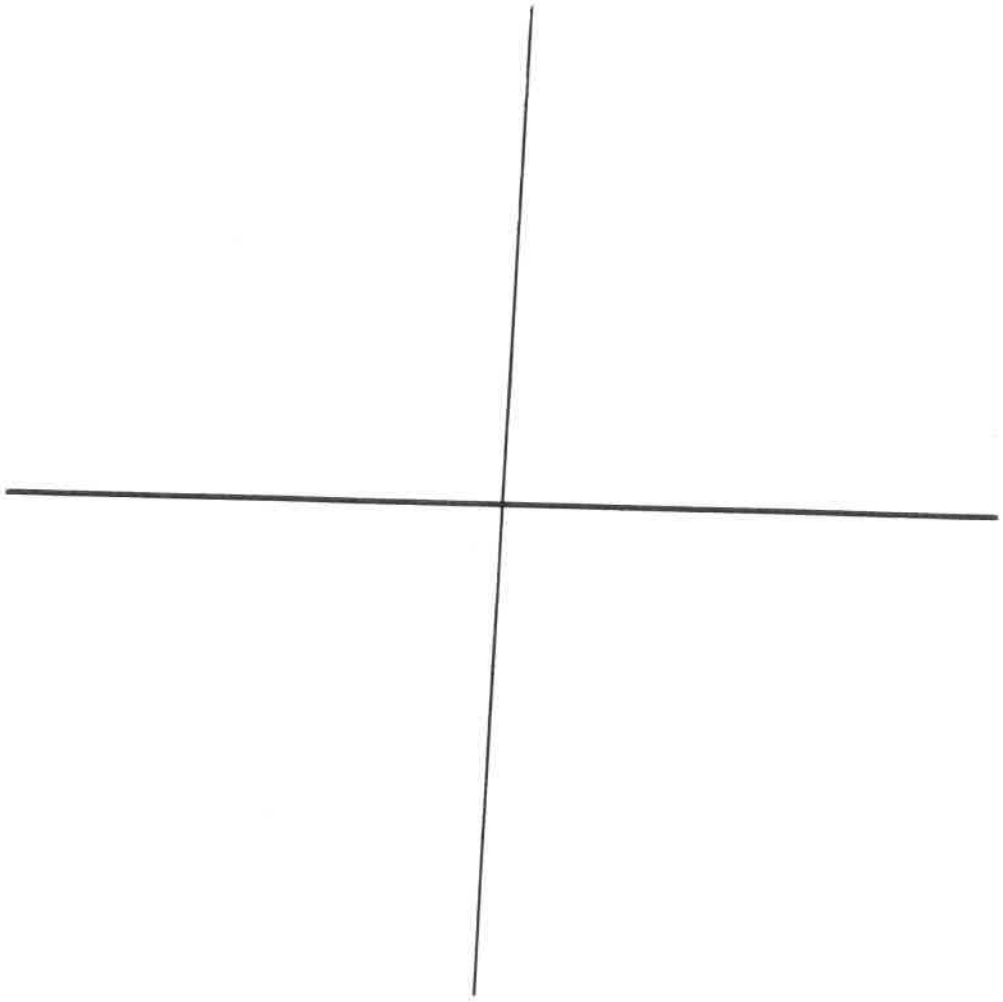
(d) Express in interval notation the set where $x^3 + x - 10 < 0$.

7. Consider the rational function $f(x) = \dfrac{2x+4}{x-5}$.

(a) Find the x and y intercepts of its graph.

(b) Give equations for any horizontal and vertical asymptotes of its graph, if they exist.

(c) Use the information obtained above to graph the function in the space below.



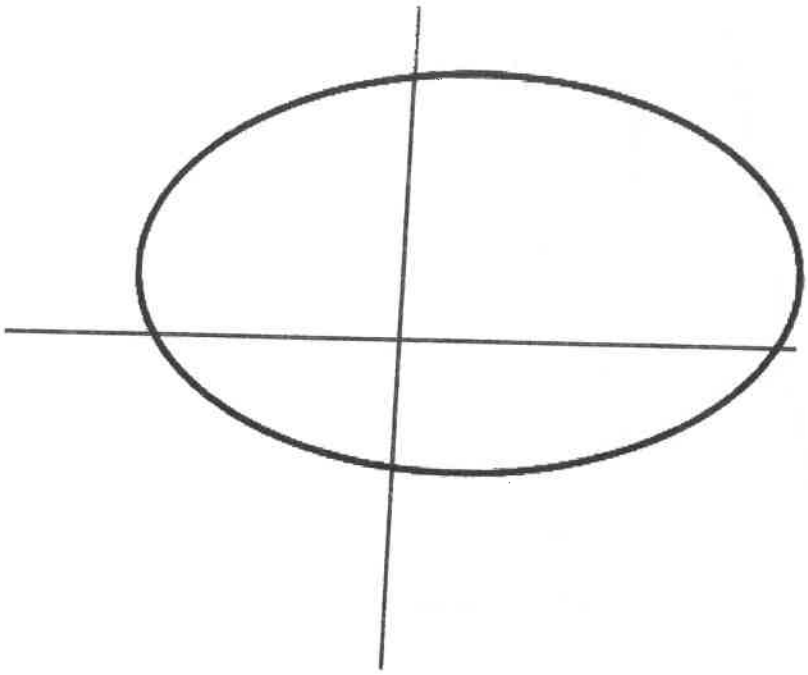
8. You are given functions $f(x) = x^2 + 4x + 1$ and $g(x) = \dfrac{-6x + 8}{8x - 3}$.

(a) If $h \neq 0$ simplify the expression $\dfrac{f(1+h) - f(1)}{h}$.

(b) Find the inverse function $g^{-1}(x)$.

(c) Find x so that $g(x) = f(-2)$.

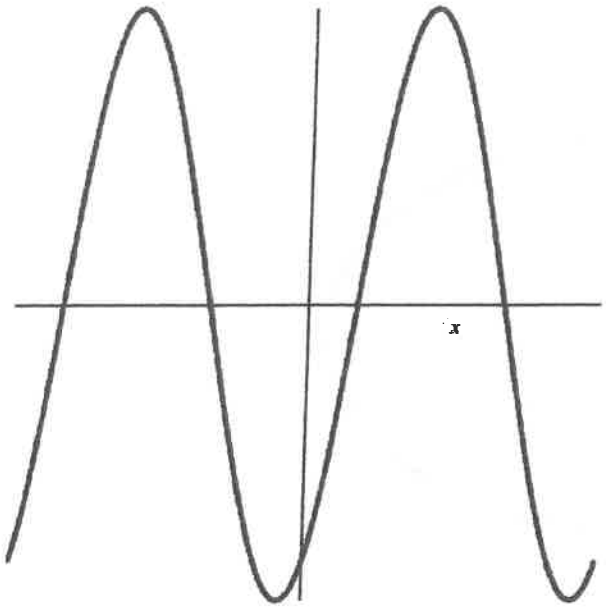
9. The ellipse shown below has center $(1, 1)$, horizontal major axis of length 10, and vertical minor axis of length 6.



(a) Give an equation of the form $\left(\frac{x-h}{a}\right)^2 + \left(\frac{y-k}{b}\right)^2 = 1$ for the ellipse.

(b) Where does the ellipse intersect the y axis? (Give coordinates of intersection points.)

10. Shown below is the graph of $f(x) = 5 \sin\left(2x - \frac{\pi}{3}\right)$ over the interval $[-\pi, \pi]$.



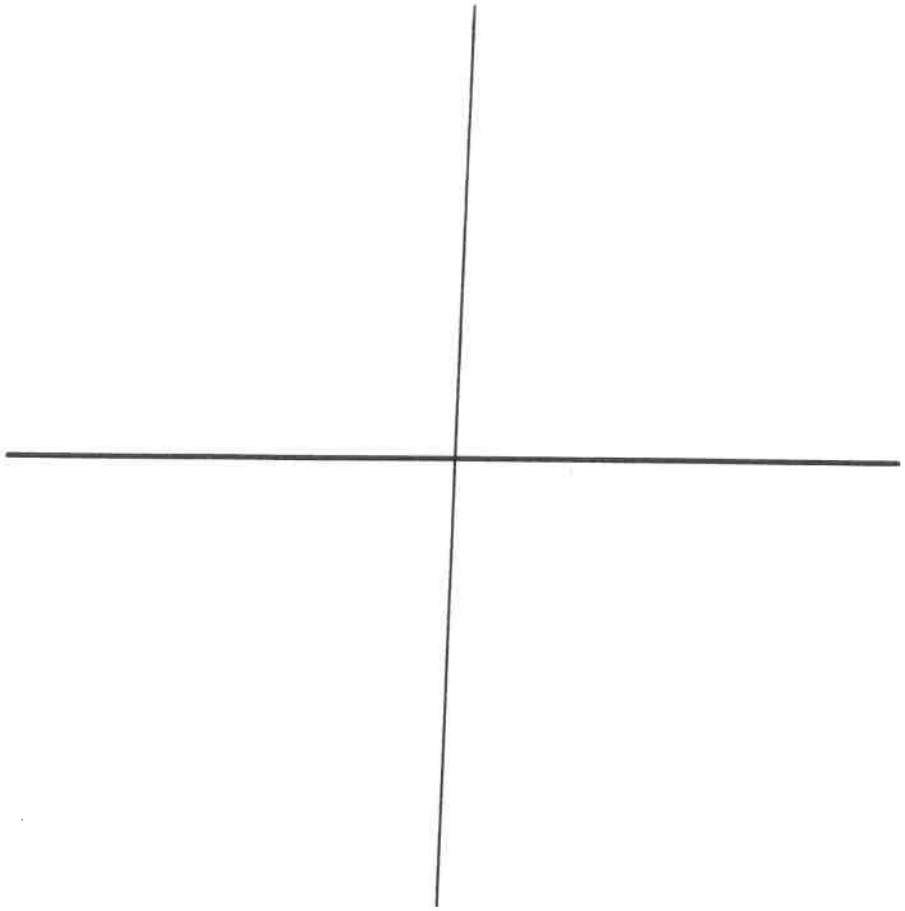
(a) What is the amplitude of this function? What is the period?

(b) Determine the y intercept of the graph. Identify it by labeling with coordinates.

(c) Determine the least positive $x \in \mathbb{R}$ for which $f(x) = 0$.

11. Consider the function $f(x) = 2^{1-x} - 4$.

(a) Plot the graph of f.



(b) Find the x and y intercepts of the graph. Identify them by labeling with coordinates.

(c) The graph of f has a horizontal asymptote. Include it in the plot with a dashed line, and label it with a defining equation.