DEPARTMENT OF MATHEMATICS
BROOKLYN COLLEGE
FINAL EXAMINATION—SPRING 2013
MATHEMATICS 1011 (PRE-CALCULUS)

Name:______________________________

PART I: Answer all 6 questions. Each question is worth 10 points. Justify each answer and show all your work.

1. (a) Let \( f(x) = x^2 + 2x - 7 \). Find and simplify \( \frac{f(x+h)-f(x)}{h} \).

(b) Solve the equation \( \left(\frac{1}{3}\right)^{x+2} = 81 \).

2. (a) Find an equation of the straight line that passes through the point \((-2, -7)\) and parallel to the line with equation \(3x - 2y = 30\).

(b) Find the center and radius of the circle with equation \(x^2 + y^2 - 2x - 6y + 1 = 0\)
and sketch its graph showing its center and at least four points on it.

3. (a) Sketch the graph of \( y = x^2 - 6x + 8 \).
Identify its vertex and \( x \) and \( y \) intercepts on the graph, and label each of these coordinate points.

(b) Write \( \frac{8 + 3i}{9 - 2i} \) in the form \( a + bi \), where \( a \) and \( b \) are real numbers.

4. (a) For all real values, find the domain of \( g(x) = \sqrt{16 - 2x} \).

(b) Solve the inequality \( x^2 - 7x + 6 \geq 0 \). Write the solution using interval notation and sketch it on the real number line.

5. (a) Find all zeros (roots), real or complex, of the polynomial \( P(x) = x^3 - 2x^2 - 5x + 6 \) and give the complete factorization of \( P(x) \).

(b) Find the exact value of \( \log_2 32 + 8^{-\frac{3}{4}} \).

6. For the function given by \( f(x) = \frac{3x^2}{4-x^2} \):
(a) Find the \( x \) and \( y \) intercepts of the graph of \( f(x) \).
(b) Find the vertical and horizontal asymptotes of \( f(x) \).
(c) Use the information above to sketch the graph of \( f(x) \).

Please turn over!
PART II: Answer 4 out of 5 questions. Each question is worth 10 points. Justify each answer and show all your work.

7. (a) Solve for $x$ and check: $\log_5(x - 4) + \log_5 x = 1$.
(b) Given $f(x) = \frac{1}{x^3 - 3}$ and $g(x) = \sqrt[3]{x^2 + 3}$. Find and simplify $f \circ g(x)$.

8. (a) Verify the identity $\cos 3x = (1 - 4 \sin^2 x) \cos x$.
(b) Find the fifth term of the binomial expansion of $(2x - 7)^6$.

9. (a) Find the center, vertices, and foci of the ellipse
\[ \frac{(x-2)^2}{9} + \frac{(y+1)^2}{16} = 1 \]
and sketch its graph showing these points.
(b) Find the exact value of $\cos[\sin^{-1}\left(\frac{2}{3}\right)]$.

10. (a) Find the amplitude, period, and phase shift of $y = 3 \sin 2(x - \frac{\pi}{4})$, and graph one complete period.
(b) Find the inverse, $f^{-1}(x)$, of the function $f(x) = \frac{1+3x}{5-2x}$.
What is the domain of $f^{-1}(x)$?

11. (a) Find all real solutions of the equation $x - \sqrt{x + 3} = \frac{x}{2}$ and check.
(b) Use the logarithmic properties to write the expression $\ln\left(\frac{(ab)^{\frac{1}{2}}}{\sqrt{c}}\right)$ as a sum or difference of simpler logarithms.

End of Examination