DEPARTMENT OF MATHEMATICS
BROOKLYN COLLEGE
FINAL EXAMINATION—SPRING 2014
MATHEMATICS 1011 (PRE-CALCULUS)

Name:

PART I: Answer all 6 questions. Each question is worth 10 points. Justify each answer and show all your work.

1. (a) Let \( f(x) = -x^2 - x + 8 \). If \( h \neq 0 \), express \( \frac{f(x+h) - f(x)}{h} \) as a polynomial in \( x \) and \( h \).
   
   (b) Find all \( x \in \mathbb{R} \) satisfying \( \left( \frac{27}{8} \right)(x-1) = \frac{9}{4} \).

2. (a) Find an equation of the straight line passing through the point \((3, -2)\) and parallel to the line with equation \(-3x + 4y = 2\).
   
   (b) Find the center and radius of the circle with equation \(2x^2 + 2y^2 - 8x + 12y + 18 = 0\). Sketch the circle, and indicate its center and at least four points on it.

3. (a) Sketch the graph of \( y = f(x) = x^2 - 2x - 3 \). Indicate its vertex and \( x \) and \( y \) intercepts. Find the domain and range of \( f(x) \).
   
   (b) On what interval in the domain is \( f(x) \) increasing? decreasing?

4. (a) Find the domain of \( g(x) = \sqrt{4x - x^2} \). Write the solution using interval notation and graph it on the real number line.
   
   (b) Use the graph of \( y = \sqrt{x} \) and transformations to sketch the graph of \( y = -\sqrt{x} + 9 \). Label all intercepts.

5. (a) Find all zeros (roots), real or complex, of the polynomial \( P(x) = x^3 - 5x^2 + 2x + 8 \). Give the complete factorization of \( P(x) \).
   
   (b) Solve for \( b \): \( \log_b \frac{1}{a} = -\frac{3}{2} \).

6. For the function given by \( f(x) = \frac{x-1}{x} \):
   
   (a) Find the \( x \) and \( y \) intercepts (if any) of the graph of \( f(x) \).
   
   (b) Find the vertical and horizontal asymptotes of \( f(x) \).
   
   (c) Use the information above to sketch the graph of \( f(x) \).

Please turn over!
PART II: Answer 4 out of 5 questions. Each question is worth 10 points. Justify each answer and show all your work.

7. (a) Solve for $x$ and check: $2 \log_{25}(x) - \log_{25}(25 - 4x) = \frac{1}{2}$

(b) Let $f(x) = \frac{1}{x^2 - 2}$ and $g(x) = \sqrt{x + 3}$. Find $(f \circ g)(x)$ and state its domain. Find $(f \circ g)(\frac{1}{4})$.

8. (a) Verify the identity: $\tan x + \cot x = \csc x \sec x$

(b) Find the sixth term in the binomial expansion of $(a + 2b)^6$.

9. (a) Find the center, vertices, and foci of the hyperbola

\[
\frac{(x+1)^2}{16} - \frac{(y-3)^2}{9} = 1.\]

Sketch its graph showing these points. Give the equations of the asymptotes.

(b) Without using a calculator, find the value of $\sin[\arccos(\frac{13}{15})]\]

(express your answer as a rational number).

10. (a) Find the amplitude, period, and phase shift of $y = 2 \cos (2x - \frac{\pi}{3})$.

Sketch its graph in one complete period.

(b) Find the inverse, $f^{-1}(x)$, of the function $f(x) = x^2 - 1, \ x \geq 0$.

What is the domain of $f^{-1}(x)$? Sketch the graph of $f^{-1}(x)$.

11. (a) Find all $x \in \mathbb{R}$ satisfying the equation $\sqrt{3x - 2} = 3 - \sqrt{3x + 1}$ and check the validity of your answer.

(b) Find $\sin(\alpha + \beta)$ and $\cos(\alpha + \beta)$, where $\alpha$ is in quadrant III with $\sin(\alpha) = -\frac{1}{3}$ and $\beta$ is in quadrant IV with $\cos(\beta) = \frac{2}{5}$.

End of Examination