PART 1: Complete the following six multipart problems, each worth 10 points. Justify your answers by showing your work.

1. A parabola has equation \( y = -2x^2 - 8x + 24 \).

   (a) Find the \( x \) and \( y \) intercepts of this parabola.

   (b) Where is vertex of the parabola?

   (c) The parabola is shown on the axis system below. Label the vertex, as well as the \( x \)- and \( y \)-intercepts, with coordinates on the plot.
2. A circle has diameter with endpoints $P(-5, 4)$ and $Q(11, -8)$.
   (a) Give the coordinates of the center of the circle.
   
   (b) What is the radius of the circle?

3. You are given the two functions $f(x) = 4x^2 - 4x - 15$ and $g(x) = \sqrt{x}$.
   (a) Simplify the expression $f(2 + h) - f(2)$ to a polynomial in $h$, assuming $h \neq 0$.
   
   (b) Evaluate $g \circ f(-2)$.

   (c) Evaluate $f \circ g(9)$?
4. Study the rational function \( f(x) = \frac{10 - 4x}{2x - 3} \).

(a) Find all \( x \in \mathbb{R} \) at which \( f(x) \) is undefined.

(b) Find all \( x \in \mathbb{R} \) at which \( f(x) = 0 \).

(c) Determine intervals of \( \mathbb{R} \) on which \( f(x) > 0 \), and on which \( f(x) < 0 \). Illustrate this information on a number line.

(d) Give equations for each horizontal or vertical asymptote (if they exist).

(e) Use the above information to graph \( f \) on the axes below.
5. Consider the graph of the function \( f(x) = 3 \sin \left( \frac{x}{2} - \frac{\pi}{6} \right) \) given below for \( x \in [-3\pi, 3\pi] \).

(a) Find the coordinates of the \( y \) intercept of the curve.

(b) Evaluate \( f \left( \frac{\pi}{2} \right) \).

6. One solution of the equation \( x^3 + 2x^2 + 5x - 26 \) is \( x = 2 \). Use this information to find all solutions of the equation.
PART 2: Answer eight of the following nine problems, each worth 5 points. Justify your answers by showing your work.

1. An ellipse has vertices at \((-2, -2)\) and \((4, -2)\), and minor axis of length 4.

   (a) Where is the center of the ellipse?

   (b) Give an equation of the form \(\left(\frac{x - h}{a}\right)^2 + \left(\frac{y - k}{b}\right)^2 = 1\) (with \(a\) and \(b\) both > 0) for the ellipse.

2. The line with equations \(y = x + 7\) and the parabola with equation \(y = x^2 - 2x - 3\) are shown together below. Where do they intersect?
3. The trigonometric function \( \frac{\sin(2\theta)}{1 - \cos(2\theta)} \) is equal to one of the following:

   (a) \( \sec(\theta) \)
   (b) \( \csc(\theta) \)
   (c) \( \tan(\theta) \)
   (d) \( \cot(\theta) \)

   Identify which, and justify your claim by verifying the equality.

4. Perform the following complex arithmetic operations, expressing each as \( a + bi \) where \( a, b \in \mathbb{Q} \).

   (a) \( (8 + 5i) \cdot (-5 - 6i) \)

   (b) \( \frac{6 - 7i}{1 - 2i} \)

   (c) \( i^{773} \)
5. Shown below are the graphs of the functions $f_1(x) = \log_5(x - 3)$ and $f_2(x) = \log_5(x - 5) + 1$.

(a) Identify which graph corresponds to which function.

(b) Find the coordinates of the point of intersection of the graphs. Label this intersection on the diagram.
6. Find all $t \in \mathbb{R}$ satisfying the equation $\sqrt{4t^2 + 7t - 1} = t + 1$.

7. A rectangular hyperbola has equation $9y^2 - 16x^2 + 18y - 64x = 199$.
   (a) Where is the center of the hyperbola?
   (b) Where are the vertices of the hyperbola?
   (c) What are the slopes of the asymptotes of the hyperbola?
8. Shown below is the graph of the function $f(x) = 4^{x-1} - 8$.

(a) Determine and identify by labeling on the graph both the $x$ and the $y$ intercepts of the graph.

(b) Determine an equation for the asymptote of the graph. Add the asymptote to the plot and label with your equation.
9. Find the amplitude and period of \( f(x) = -\frac{5}{2} \cos(2x) \), and graph this function over the interval \([-\pi, \pi]\) on the coordinate system provided. Identify by labeling with coordinates all \( x \) and \( y \) intercepts.

amplitude: period: