DEPARTMENT OF MATHEMATICS
BROOKLYN COLLEGE
MATH 1201 Calculus I
Final Examination-FALL 2014

Instructions: Solve all four problems in Part I and any seven of the nine problems in Part II. Show all work for full credit. Scientific calculators permitted. No graphic calculators, cellphones, or other electronic devices permitted.

PART I: Do All Problems in This Part. (44 points)

(1) (8 points) Use the *definition* of derivative to find $f'(-2)$ for $f(x) = \sqrt{9-x^2}$. (Obtaining $f'(-2)$ using the rules of differentiation will earn *absolutely no credit*).

(2) (12 points) Find $\frac{dy}{dx}$ for each of the following.

(a) $y = e^{-3x}\sqrt[5]{x^2}$ (b) $y = (\tan x)^{\frac{1}{x}}$ (c) $y = \sec^5(x^3)$

(3) (12 points) Find the following antiderivatives.

(a) $\int \cos^3(5x)\,\sin(5x)\,dx$ (b) $\int \frac{e^{-2x}}{2-e^{-2x}}\,dx$ (c) $\int \frac{2x^2+2}{\sqrt{x^3+3x-7}}\,dx$

(4) (12 points) Let $f(x) = x^3 - 3x^2 - 9x + 12$.

(a) Find the intervals on which $f(x)$ is increasing or decreasing.
(b) Find the local maximum and local minimum values of $f(x)$, if they exist.
(c) Find the intervals of concavity and the inflection points of $f(x)$, if they exist.
(d) Carefully sketch the graph of the function f, labeling the points you found in parts (b) and (c).

PART II. Do any seven out of the nine problems in this part (56 points).

(5) Evaluate the following limits. Explain your answers.

(a) $\lim_{x\to 0} \frac{\sin(5x)\,\cos(3x)}{(x^2-2x)}$ (b) $\lim_{x\to\infty} \frac{\sqrt{4x^2-2x+5}}{3x+7}$

(6) Find area under the graph of $y = \frac{(\ln x)^3}{x}$, above the x-axis, and between the lines $x = 1$ and $x = e$.

(7) Find an equation of the tangent line to the curve $3xy^2 - 2(x+y)^3 = 4$ at the point $(2, -1)$.

(8) A ladder 15 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 2 $\frac{ft}{s}$, how fast is the angle between the ladder and the wall changing when the bottom of the ladder is 9 ft from the wall?

(9) Find the absolute maximum and absolute minimum values of $f(x) = \frac{x-2}{x^2+5}$ on the interval $[-2, 1]$.

(10) A box with a square base and open top must have a volume of $4,000\ cm^3$. Find the dimensions of the box that minimize the amount of material used.

(11) A particle moves along the x-axis with an acceleration according to the formula $a(t) = \cos t - \sin 2t$, where t is measured in seconds and displacement is measured in meters. If at time $t = 0$ the position of the particle is 3 meters (i.e., $s(0) = 3$) and the velocity is 1.5 meters per second (i.e., $v(0) = 1.5$), find the position of this particle at $t = \frac{\pi}{2}$.

(12) Consider the function $f(x) = \sqrt{x+4}$.

(a) Give an equation for the line tangent to the graph of this function at $x = 0$.
(b) Use the equation of this tangent line to find a linear approximation of $\sqrt{4.08}$.

(13) Sketch the graph of a function that satisfies **ALL** of the following conditions:

a: f is differentiable at all numbers $x \neq 3$,
b: $f(1) = 0$ and f has no other x-intercept,
c: f has a vertical asymptote at $x = 3$,
d: $\lim_{x\to\pm\infty} f(x) = -1$,
e: $f'(x) > 0$ when $x > 3$ or $-2 < x < 3$ and $f'(x) < 0$ when $x < -2$. $f'(-2) = 0$.
f: $f''(x) > 0$ when $-3 < x < 3$ and $f''(x) < 0$ when $x < -3$ or $x > 3$. $f''(-3) = 0$.