Solve all three problems in Part I and any six problems in Part II. Please show all your work.

PART I: Solve all three problems.

1. Find \( f'(x) \) for each of the following: (15 pts)

   a. \( f(x) = (e^{3x} + x^3)(2x^2 - 7)^{10} \)

   b. \( f(x) = \frac{1 + \cos(x^3)}{\sin^2 x} \)

   c. \( f(x) = (\tan 2x)^x \)
2. Evaluate each of the following integrals: (15 pts)

a. \( \int 4xe^{3x^2} \, dx \)

b. \( \int_1^4 \left( \frac{1+x}{x} \right)^2 \, dx \)

c. \( \int (5 + \sin 2x)^4 \cos 2x \, dx \)
3. Let $f(x) = 2x^3 + 3x^2 - 36x$: (10 pts)

a. Find the intervals where $f(x)$ is increasing and where it is decreasing.

b. Find the intervals where $f(x)$ is concave up and where it is concave down.

c. Find the local extremal points and the inflection points of $f(x)$.

d. Use the above information to sketch the graph of $f(x)$. 
Part II: Solve six of the following seven problems: (Each question is worth 10 points).

4. Find the absolute maximum and minimum of \( f(x) = \frac{x}{x^2+1} \) on \([0, 5]\).

5. Find an equation of the tangent line to \( x^3 + y^3 = 6xy \) at \((3, 3)\).

5. The volume of a cube is increasing at the rate of 10 cm\(^3\)/min. How fast is the surface area increasing when the length of an edge is 30 cm?
7. Find the point on the parabola \( y^2 = 2x \) that is closest to the point \((1, 4)\).

8. Find the area of the region bounded by \( y = \frac{\ln x}{x} \), \( x = 1 \) and \( x = e \).
9. A ball is thrown upwards with an initial velocity of 64 ft/sec from the top of a building which is 80 feet high. Its acceleration due to gravity is -32ft/sec^2. Determine the velocity of the ball when it hits the ground.

10. Use the definition of the derivative to find \( f'(x) \) if \( f(x) = \sqrt{3x - 7} \).