Instructions: Solve all five problems in Part I and any six of the eight problems in Part II. Show all work for full credit. Scientific calculators permitted. No graphic calculators, cell-phones, or other electronic devices permitted.

PART I: Do All Problems in This Part. (52 points)

(1) (8 points) Use the definition of derivative to find \( f'(1) \) for \( f(x) = \frac{3}{x^2 + 1} \). (Obtaining \( f'(1) \) using the rules of differentiation will earn absolutely no credit).

(2) (8 points) Evaluate the following limits. Explain your answers.

(a) \[ \lim_{x \to 1} \frac{\sqrt{x^2 + 3} + 2x}{x^2 - 1} \]

(b) \[ \lim_{x \to \infty} \frac{6x^2 - 3x + 7}{\sqrt{9x^4 + 4}} \]
(3) (12 points) Find $f'(x)$ for each of the following.

(a) $f(x) = (2x^3 + 1)^5 \sqrt{2x - 1}$

(b) $f(x) = \ln \left( \frac{2^x}{\cos^2 x} \right)$

(c) $f(x) = \sqrt{\sin \left( \frac{1}{x^2} \right)}$
(4) (12 points) Find the following antiderivatives.

(a) \[ \int \sec^3(2x) \tan^3(2x) \, dx \]

(b) \[ \int \frac{6x}{\sqrt{x^2 + 4}} \, dx \]

(c) \[ \int \frac{\cos \left( \frac{x}{2} \right)}{x^2} \, dx \]
(5) (12 points) Let \( f(x) = x^3 - 6x^2 - 15x \).

(a) Find the intervals on which \( f(x) \) is increasing or decreasing.
(b) Find the local maximum and local minimum values of \( f(x) \), if they exist.
(c) Find the intervals of concavity and the inflection points of \( f(x) \), if they exist.
(d) Carefully sketch the graph of the function \( f \), labeling the points you found in parts (b) and (c).
PART II. Do any six out of the eight problems in this part (48 points).

6) Find an equation of the tangent line to the curve \( xy^2 = 4e^{2x-y} \) at the point \((1, 2)\).

7) (a) Find the area under the graph of \( y = \frac{1}{2x+1} \), above the \( x \)-axis, and between the lines \( x = 0 \) and \( x = 3 \).

(b) Estimate the area of the region in part (a) using three approximating rectangles and right endpoints.
(8) The length of a rectangle is increasing at a rate of \( \frac{5 \text{ cm}}{s} \) and its width is increasing at a rate of \( \frac{3 \text{ cm}}{s} \). When the length is 30 cm and the width is 15 cm, how fast is the area of the rectangle increasing?

(9) Find the absolute maximum and absolute minimum values of \( f(x) = \sqrt{x}(1-x) \) on the interval \([0, 4]\).

(10) A particle moves along the \( x \)-axis with an acceleration according to the formula \( a(t) = e^{2t} \), where \( t \) is measured in seconds and displacement is measured in meters. If at time \( t = 0 \) the position of the particle is 3 meters and the velocity is 2.5 meters per second, find the position of this particle at \( t = 2 \).
(11) A farmer with 1600 $ft$ of fencing wants to enclose a rectangular area and then divide it into three rectangles with fencing parallel to one side of the rectangle (see the picture below). What is the largest possible total area of the three rectangles?

![Diagram of a rectangular area divided into three rectangles with fencing]

(12) Find all values of the constant $c$ for which $g(x)$ is continuous at all real numbers.

$$g(x) = \begin{cases} 
ce^{-x}, & \text{if } x \leq 1; \\
\frac{x}{e^x}, & \text{if } x > 1. 
\end{cases}$$
(13) Sketch the graph of a function that satisfies ALL of the following conditions:
   a: $f$ is continuous and differentiable at all numbers $x \neq 2$,
   b: $f(0) = f(4) = 0$ and $f$ has no other $x$-intercept,
   c: $f$ has a vertical asymptote at $x = 2$,
   d: $\lim_{x \to \pm\infty} f(x) = 3$,
   e: $f'(x) > 0$ when $x > 2$ or $x < -4$ and $f'(x) < 0$ when $-4 < x < 2$. $f'(-4) = 0$.
   f: $f''(x) > 0$ when $x < -6$ and $f''(x) < 0$ when $-6 < x < 2$ or $x > 2$. $f''(-6) = 0$. 
Name: ______________________

Department of Mathematics
Brooklyn College
Final Examination – Fall 2013
Math 1201: Calculus I
Total Points 100

1) [5 Points] Find the derivative of \( f(x) = \sqrt{1 - 6x} \).

2) [7 Points each] Find the derivative of:
   a) \( y = x^2 \sqrt{9 - x} \)
   b) \( y = \sec \sqrt{x^2 + 1} \)
   c) \( y = (\cos 2x)^3 \)
   d) \( x^4 - 2xy + e^{3x} = 2 \)

3) [24 Points] Find the following integrals:
   a) \( \int \frac{x + 1}{(x^2 + 2x + 3)^2} \, dx \)
   b) \( \int e^{6x} \sec^2 4x \, dx \)
   c) \( \int \frac{(\ln x)^2}{x} \, dx \)
   d) \( \int_0^\pi \cos^2(2x)\sin(2x) \, dx \)

4) [24 Points] For the function \( f(x) = x^3 + 6x^2 + 9x + 3 \)
   a) Find the intervals over which the function is increasing and decreasing;
   b) Find the local extrema;
   c) Find the inflection points and discuss its concavity; and
   d) Draw the graph clearly indicating the y-intercept, local extrema, and inflection points.

5) [7 Points] A ball is thrown upwards with an initial velocity of 48 feet/sec from the top of a 160 foot tall building. Assuming that the only force affecting the ball during travel is from gravity, which produces a downward acceleration of 32 feet/sec\(^2\), find the velocity of the ball when it hits the ground.

6) [7 Points] Two cars, initially 130 miles apart, start off at the same time. Car A is to the west of Car B and starts traveling east (towards Car B) at 25 miles per hour while at the same time Car B starts traveling south at 10 miles per hour. After 4 hours of traveling at what rate is the distance between the two cars changing? Is it increasing or decreasing?

7) [7 Points] Find the point (or points) on the curve \( x^2 - y^2 = 16 \) which is (are) closest to the point (0,8). Justify your answer.

8) [7 Points] Find an equation of the tangent line to the graph of \( y = x \ln x \) at the point where \( x = e^3 \).