PART I: Do All Problems in This Part. (52 points)

(1) Evaluate the following: (18 points)

(a) \[ \int (4x - 7) e^{3x} \, dx \]

(b) \[ \int \frac{7x^2 + 2x + 25}{(x^2 + 9)(x + 1)} \, dx \]
(2) Determine whether the series is absolutely convergent, conditionally convergent, or divergent. Explain your answers by stating the test you are using and why the conditions of the test are satisfied. (20 points)

\[ \sum_{n=1}^{\infty} (-1)^n \left( \frac{2n + 4}{3n - 1} \right)^n \]

(a) \[ \sum_{n=1}^{\infty} (-1)^n \left( \frac{2n + 4}{3n - 1} \right)^n \]
(b) \[ \sum_{n=1}^{\infty} \frac{5n^2 - 1}{\sqrt{n^6 + 2n^3 + 4}} \]

(c) \[ \sum_{n=1}^{\infty} \frac{\cos n}{n^2 + 1} \]

(d) \[ \sum_{n=1}^{\infty} \frac{(-1)^n}{n \sqrt{\ln n}} \]
(a) The area of $\mathcal{R}$.

(b) The volume of the solid generated by revolving $\mathcal{R}$ about the $x$-axis.

(c) The volume of the solid generated by revolving $\mathcal{R}$ about the $y$-axis.

(d) The length of the part of the curve $y = x^2 - 6x + 11$ that bounds $\mathcal{R}$. 
PART II. Do any four out of five problems in this part (48 points).

(4) Find the radius of convergence and interval of convergence of the series. Remember to check the endpoints.

\[ \sum_{n=1}^{\infty} \frac{(-3)^n(2x - 1)^n}{\sqrt{n + 1}} \]
(b) If $f(x) = e^{\arctan x} + \arcsin x^2$, find $f'(x)$. 
(6) (a) Explain why the integral below is improper and determine if it is convergent or divergent. Evaluate the integral if it is convergent.

\[ \int_0^2 \frac{dx}{\sqrt{4 - x^2}} \]

(b) Find the sum of the series

\[ \sum_{n=2}^{\infty} \frac{5^{n-1}}{23n-4} \]
(b) Find the 3rd degree Taylor polynomial for \( f(x) = \frac{1}{\sqrt{5-2x}} \) about \( x = 2 \).
(8) Given the power series \( e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \).

(a) Find a power series for \( \int x^2 e^{-3x^2} \, dx \).

(b) Express \( e^{-\frac{x}{3}} \) as an infinite series and determine how many terms of your series are needed to approximate \( e^{-\frac{x}{3}} \) accurate within an error of at most \( 10^{-4} \). Show all work.