Math 1206  Final Examination  Fall 2013  
Department of Mathematics, Brooklyn College  
Show all work and justify all your answers  

PART I (40 points) : Answer all questions in this part.  

1. (15 points)  
Determine if the following series converge or diverge:  

\[ a) \sum_{n=1}^{\infty} n^2 3^{-n} , \quad b) \sum_{n=1}^{\infty} \frac{(-1)^n n^2}{n^3 + 1} , \quad c) \sum_{n=2}^{\infty} \frac{1}{n (\ln n)^2} . \]

2. (18 points)  
Evaluate each of the following integrals:  

\[ a) \int \frac{(2x^2 - x + 1)}{x^3 + x} \, dx , \quad b) \int \tan^3 x \sec^4 x \, dx , \quad c) \int_{0}^{2} \frac{dx}{(x^2 + 4)^{3/2}} . \]

3. (7 points)  
Determine if the following improper integral converges or diverges and find its value if it converges.  

\[ \int_{0}^{\infty} x^3 e^{-3x} \, dx . \]

PART II (60 points) : Answer any four (4) of the five (5) questions in this part.  

4. (15 points)  
\[ a) \text{Sketch the graph of } r = \cos(3\theta) \text{ and find the area of one leaf of the curve.} \]
\[ b) \text{Determine if the series} \sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n}}{\sqrt{n^3 + 3}} \quad \text{is absolutely convergent, conditionally convergent, or divergent.} \]
5. (15 points)
a) State the McLaurin series for $e^x$ and use it to obtain the McLaurin series for $e^{-x^2}$. Be sure to state the general term of the series. Estimate the value of the integral $\int_0^{0.2} e^{-x^2} dx$ to four decimal place accuracy by using the above series.

b) Find the first four terms of the Taylor series expansion of $f(x) = \sqrt{x + 3}$ around $x = 1$.

6. (15 points)
a) Find the area between the curve $y = \cos^3(x)$ and the x-axis from $x = \pi/4$ to $x = \pi/3$.

b) Let $V$ denote the volume obtained by rotating around the X-axis the area bounded by $y = 2x^2$ and $y = 2$.

Set up (but do not evaluate) the formulas for $V$ using the disc (or washer) method and the cylindrical shell method.

7. (15 points)
a) Find $dy/dx$ for the following:

$$y = \tan^{-1}(\sqrt{x - 2}) + \sqrt{\sin^{-1} x} .$$

b) Find the arc length of the curve

$$y = (e^x + e^{-x})/2 , \ -1 \leq x \leq 1 .$$

8. (15 points)
a) Find the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(2x - 3)^n}{n} .$$

Find the power series obtained from the above series by term by term differentiation and find its interval of convergence.

b) Evaluate

$$\lim_{x \to 0} (1 + 3x)^{\frac{1}{x}} .$$

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