INSTRUCTIONS: Answer all questions in Part I and any four questions in Part II. Show all work and justify all your answers.

PART I (52 points): Answer all questions in this part.

(18 pts) 1. Determine if each of the following series converges or diverges:

(a) \[ \sum_{n=1}^{\infty} n^{-3} \ln n \]

(b) \[ \sum_{n=1}^{\infty} \frac{(-1)^n n^3}{n^3 + 1} \]
(16 pts) 3. Consider the region $R$ pictured to the right. Set up (but do not evaluate) definite integrals for each of the following. Always integrate along the $x$-axis.

(a) the area of $R$,

(b) the volume of the solid that results when $R$ is rotated about the $x$-axis,
(c) the volume of the solid that results when $R$ is rotated about the $y$-axis,

(d) the length of the portion of the parabola that bounds $R$. 
PART II (48 points): Answer any three of the four questions in this part. Each problem is worth 16 points.

4. Indicate where the series \( \sum_{n=0}^{\infty} \frac{(2x-1)^n}{3^n \sqrt{n+1}} \) is (i) absolutely convergent, where it is (ii) conditionally convergent, and where it is (iii) divergent. Justify your answers.
5. (a) Set up (but do not evaluate) a definite integral that represents the area of the region \( R \) inside the curve \( r = 3 \cos \theta \) and outside the curve \( r = 2 - \cos \theta \). Carefully sketch the region \( R \).

(b) Find the Taylor polynomial of degree three for \( f(x) = \frac{1}{(x-2)^3} \) about \( x = 3 \).
6. (a) (i) Determine if the improper integral \( \int_{0}^{\infty} xe^{-z/2} dz \) is convergent or divergent. (ii) If it is convergent, find the value of the integral.

(b) Find \( f'(x) \) if \( f(x) = \arctan 3x + \ln(\arccos 5x) \).
7. (a) Using the Maclaurin series $\cos x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!}$, (i) find the Maclaurin series of the integral $\int \cos(x^2) \, dx$. (ii) Estimate the value of the integral $\int_{0}^{0.4} \cos(x^2) \, dx$ to four decimal place accuracy by using the series obtained in part (i).

(b) Evaluate the limit $\lim_{x \to 0} \frac{e^{-x} + \sin 4x}{x^{3/2}}$. 

END OF EXAMINATION