INSTRUCTIONS: Answer all questions in Part I and any four questions in Part II. Show all work and justify all your answers.

PART I (40 points): Answer all questions in this part.

(15 pts) 1. Determine if each of the following series is absolutely convergent, conditionally convergent, or divergent.
   
   (a) \[ \sum_{n=1}^{\infty} (-1)^n n^3 2^{-n} \]
   
   (b) \[ \sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2 + 1} \]
   
   (c) \[ \sum_{n=2}^{\infty} \frac{(-1)^n}{n \sqrt{\ln n}} \]

(18 pts) 2. Evaluate each of the following integrals:

   (a) \[ \int \frac{x^2 + x + 4}{x^3 + 4x} \, dx \]
   
   (b) \[ \int \tan^3 x \sec^5 x \, dx \]

   (c) \[ \int_{0}^{2} \frac{dx}{(x^2 + 4)^{3/2}} \]

(7 pts) 3. Determine if the following improper integral converges or diverges and find its value if it converges:

   \[ \int_{0}^{\infty} xe^{-2x} \, dx \]
PART II (60 points): Answer any four of the five questions in this part. Each question is worth 15 points.

4. (a) Sketch the graph of \( r = \cos 3\theta \) and find the area of one leaf of the curve.

(b) Determine if the following improper integral converges or diverges and find its value if it converges:
\[
\int_0^2 \frac{dx}{\sqrt{4 - x^2}}
\]

5. (a) Use the McLaurin series of \( e^x \) to find the McLaurin series of \( \int_0^{0.2} e^{-x^2} \ dt \) (give the general term of the series). Estimate the value of the integral \( \int_0^{0.2} e^{-x^2} \ dx \) to four decimal place accuracy by integrating a suitable series.

(b) Find the first four terms of the Taylor series expansion of \( f(x) = \sqrt{x} \) around \( x = 1 \).

6. (a) Find the area under the curve \( y = \sin^4 x \cos x \) and above the \( x \)-axis from \( x = \pi/6 \) to \( x = \pi/3 \).

(b) Let \( V \) denote the volume obtained by rotating around the \( x \)-axis the area bounded by \( y = x^2 \) and \( y = 4 \). Set up (but do not evaluate) the formulas for \( V \) using (i) the disc (or washer) method and (ii) the cylindrical shell method.

7. (a) Find \( dy/dx \) if \( y = \arctan \sqrt{x} + \sqrt{\arccsc x} \).

(b) Find the arc length of the curve \( y = (e^x + e^{-x})/2 \), \(-1 \leq x \leq 1 \).

8. (a) Find the interval of convergence of the power series \( \sum_{n=1}^{\infty} \frac{(3x - 2)^n}{n4^n} \). Find all points in the interval of convergence where the series is absolutely convergent. State clearly the theorems you use in your work.

(b) Evaluate \( \lim_{x \to 0} (1 - 2x)^{1/x} \).

END OF EXAMINATION