DEPARTMENT OF MATHEMATICS
BROOKLYN COLLEGE

FINAL EXAMINATION – SPRING 2015
MATHEMATICS 1206

INSTRUCTIONS: *Answer* **all** *questions in Part I and any* **three** *questions in Part II.* **Show all work** *and* **justify all your answers.**

PART I (52 points): *Answer all questions in this part.*

(21 pts) 1. Evaluate each of the following integrals:

(a) $\displaystyle\int \frac{x^2 - 7x - 12}{(x-1)(x+2)^2}\,dx$ (b) $\displaystyle\int_0^{\pi/2} x \sin 2x\,dx$

(c) $\displaystyle\int \frac{1}{x^2\sqrt{x^2-9}}\,dx$

(15 pts) 2. Determine if each of the following series is (i) absolutely convergent, (ii) conditionally convergent, or (iii) divergent:

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\, 5^n \cdot n^3}{(n+1)!}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\, n^2}{\sqrt{n^4+1}}$

(c) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n\, n^2}{(n^3+1)(\ln n)^2}$

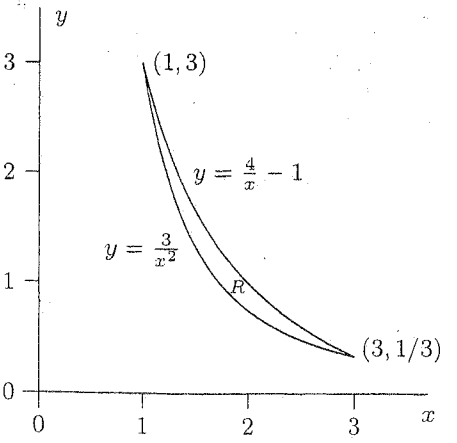
(16 pts) 3. Consider the region R pictured to the right. Set up (but *do not evaluate*) definite integrals for each of the following. Always integrate along the x-axis.

(a) the area of R,

(b) the volume of the solid that results when R is rotated about the x-axis,

(c) the volume of the solid that results when R is rotated about the y-axis,

(d) the length of the boundary (i.e., the perimeter) of R.



All computer processing for this document was done under Linux. $\mathcal{AMS}$-TEX was used for typesetting. PICTEX was used for the diagram, together with the programming language Perl for generating the data points. The computer algebra system Maxima was used in designing some of the problems.

(continued)

PART II (48 points): *Answer any* **three** *of the* **four** *questions in this part. Each problem is worth 16 points.*

4. Indicate where the series $\sum_{n=1}^{\infty} \frac{(2x+5)^n}{7^n \sqrt{n}}$ is (i) absolutely convergent, where it is (ii) conditionally convergent, and where it is (iii) divergent. Justify your answers.

5. (a) Set up (but *do not evaluate*) a definite integral that represents the area of the region R inside the curve $r = 3 + 2\sin\theta$ and outside the curve $r = 5\sin\theta$. Carefully sketch the region R.

   (b) Find $f'(x)$ if $f(x) = \arccos(4x^2 - 1) + x^2 \arctan(2x + 3)$.

6. (a) (i) Determine if the improper integral $\int_0^{\infty} \frac{x}{(x^2+2)^3}\, dx$ is convergent or divergent. (ii) If it is convergent, find the value of the integral.

   (b) Find the Taylor polynomial of degree three for $f(x) = \ln(x - 4)$ about $x = 5$.

7. (a) Using the Maclaurin series $\sin x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}$, (i) find the Maclaurin series of the integral $\int \frac{\sin x}{x}\, dx$; make sure to give the *general term* of the series, not just the first few terms. (ii) Estimate the value of the integral $\int_0^{0.2} \frac{\sin x}{x}\, dx$ to four decimal place accuracy by using the series obtained in part (i).

   (b) Evaluate the limit $\lim_{x \to 0} (\sin 3x + \cos 2x)^{2/x}$.

END OF EXAMINATION