Instructions. This exam consists of two parts. You must do all the problems in part I and seven of the nine problems in part II. You must show all your work and cross out any work you do not want graded. You may use a scientific calculator, but are not allowed to use any notes or books. By signing your blue book, you are pledging that you have neither given nor received assistance on this exam. An answer without an explanation will receive no credit. Please simplify your answers as much as possible and box your final answers.

Good luck!

Part I Do all three problems in this part.

1. (18 points) Evaluate the following:
   (a) \( \int_{0}^{\pi/2} \cos^5(x) \, dx \)
   (b) \( \int_{0}^{2} \frac{dx}{(x^2 + 4)^2} \) [Hint: Use an appropriate trigonometric substitution.]
   (c) \( \int \frac{x - 4}{(x + 1)(x^2 + 1)} \, dx \)

2. (18 points) Determine for each of the following series if it is absolutely convergent, conditionally convergent, or divergent. Give a complete justification of your answer.
   (a) \( \sum_{n=1}^{\infty} (-1)^n \frac{n}{\sqrt{4 + n^5}} \)
   (b) \( \sum_{n=2}^{\infty} (-1)^n \frac{1}{n \ln(n)} \)
   (c) \( \sum_{n=2}^{\infty} (-1)^n \left( \frac{n - 1}{n^2} \right)^n \)

3. (8 points) Determine if the integral
   \( \int_{0}^{\infty} (1 + 3x)e^{-x} \, dx \)
   converges or diverges. If it converges, compute its value.
Part II Do any 7 of the 9 problems in this part.

4. (8 points) Use the fact that
\[ \frac{1}{1 + x^2} = \sum_{n=0}^{\infty} (-1)^n x^{2n} \]
for \(|x| < 1\) to do the following:

(a) Find a power series representation for \(\arctan x\) valid for all \(x\) with \(|x| < 1\).
(b) Find a power series representation, valid for all \(x\) with \(|x| < 1\), for
\[ \int \arctan(x^2) \, dx. \]

5. (8 points) Use the fact that for \(|x| < 1\),
\[ \ln(1 + x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{n} \]
to evaluate \(\int_0^{1/2} \ln(1 + x^2) \, dx\), correct to within an error of 0.001.

6. (8 points) Consider the region bounded by the curves \(y = \ln x\), the line \(y = 2\), \(y = 3\), and the \(y\)-axis. Compute the volume of the solid obtained by rotating the region about the \(y\)-axis. The graph of \(y = \ln x\) is given here in case it is helpful.

7. (8 points) Find the arclength of the curve given by \(f(x) = \frac{4}{3} x^{3/2}\) for \(2 \leq x \leq 6\).

8. (8 points) Find the Maclaurin series for \(f(x) = \frac{\sin(2x)}{x}\). Give your answer in summation notation. [Hint: Find first the Maclaurin series for \(\sin(2x)\).]

9. (8 points) Find the radius and interval of convergence for
\[ \sum_{n=1}^{\infty} \frac{5^n x^n}{(n + 8)^3}. \]
10. (8 points) Use L'Hôpital's Rule to compute
\[ \lim_{x \to 1} \frac{(\ln x)^2}{x - 1}. \]

11. (8 points) Use either the trapezoidal rule or the midpoint rule with \( \Delta x = \pi/6 \) to estimate
\[ \int_{-\pi/3}^{\pi/3} \sec^4 x \, dx. \]
State clearly which rule you are using. Is this sum an overestimate or an underestimate? Why? [Hint: The picture below may be helpful.]

12. (8 points) Find the area enclosed by the curve with equation \( r^2 = 3 \sin(2\theta) \), depicted below.