This is a collection of questions that will be representative of those on the final exam. This is **NOT** a sample final exam: the final exam may have more or fewer questions, or the questions may be asked in a different format.

There **WILL** be other types of questions on the final exam. However, about 80% of your grade on the exam will be based on questions similar to the following. (The remainder will be based on everything else you did in the course)

Limits, Continuity, Definition of the Derivative

Question L1

Suppose $\lim_{x \to 3} f(x) = 12$, $\lim_{x \to 3} g(x) = -8$. Find the following.

$$\lim_{x \to 3} f(x)g(x)$$

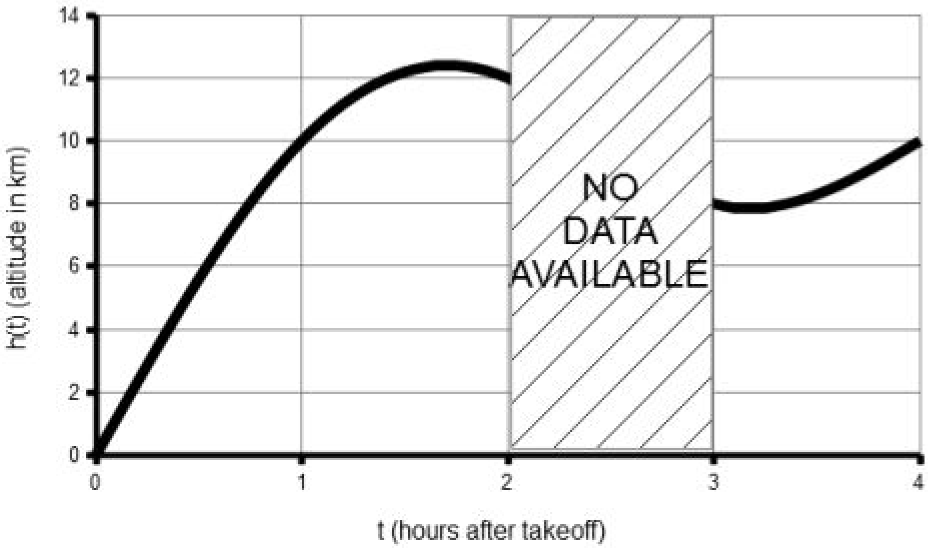
$$\lim_{x \to 3} \frac{12 - f(x)}{4 + g(x)}$$

$$\lim_{x \to 3} \frac{\sin(12 - f(x))}{12 - f(x)}$$ (this might not always exist)

$$\lim_{x \to 3} \left([f(x)]^3 - 6g(x) + x^2\right)$$

Question L3

You get on an airplane in Los Angeles. The plane leaves at 9 AM, and arrives in New York six hours later; note that both cities are at sea level. Let h(t) be the altitude of the plane, in kilometers above sea level, t hours after 9 AM. Due to a malfunction in the flight data recorder, possibly caused when the plane flew over Area 51, some of the data was erased. A graph of the data is shown below:



Use the graph and your knowledge of the real universe to answer the following questions.

Is h(t) a continuous function? Why/why not?

If possible, find $\lim_{t \to 2^-} h(t)$.

If possible, find $\lim_{t \to 3^-} h(t)$.

Find h(6). (No, the "6" is not a typo or a mistake in the question)

Find $\lim_{t \to 6} h(t)$.

Find the average rate of change of the altitude over the interval from t = 0 to t= 6.

True or false: At some point between t = 2 and t = 3, the plane's altitude was exactly 8.275 km.

Which of the following could be used to answer the question "How rapidly did the plane descend during its last half hour in the air?

(a) h(6) - h(5.5)

(b) $\dfrac{h(6) - h(5.5)}{6 - 5.5}$

(c) 6 - 5.5

(d) This question cannot be answered with the given information.

Which of the following could be used to answer the question "How far did the plane descend during its last half hour in the air?

(a) h(6) - h(5.5)

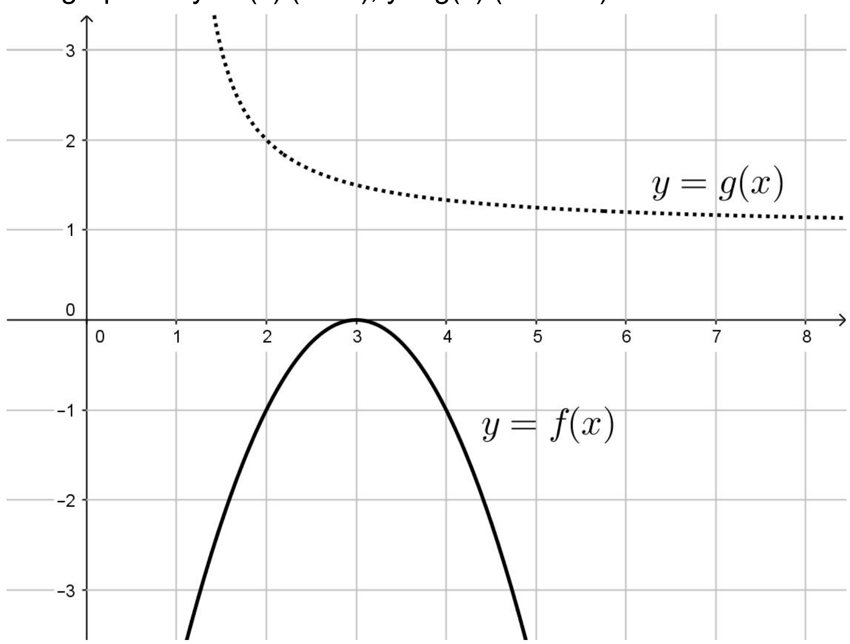
(b) $\dfrac{h(6) - h(5.5)}{6 - 5.5}$

(c) 6 - 5.5

(d) This question cannot be answered with the given information.

<u>Question L4</u>

The graphs of y = f(x) (solid), y = g(x) (dashed) are shown.



Find the following.

$$\lim_{x \to 2} \frac{g(x)}{f(x)}$$

$$\lim_{x \to 3} f(x)$$

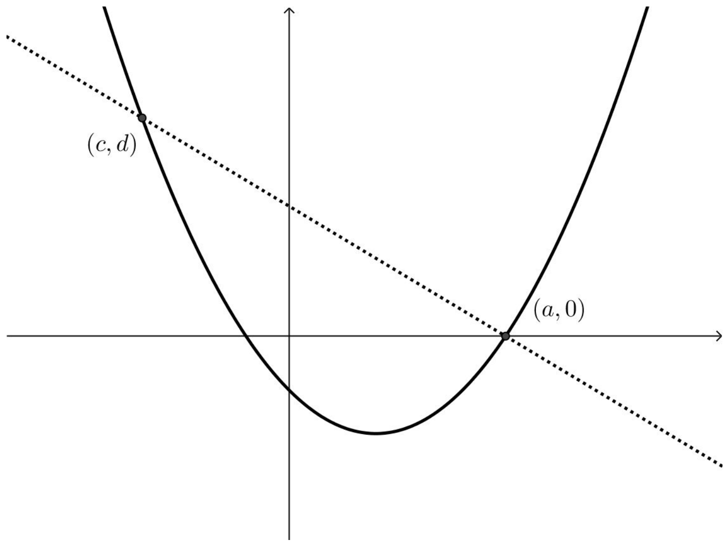
$$\lim_{x \to 3} \frac{1}{f(x)}$$

$$\lim_{x \to \infty} g(x)$$

$$\lim_{x \to \infty} \frac{g(x)}{f(x)}$$

Question L5

The graphs of y = f(x) (solid) and y = g(x) (dotted) are shown.



Find the following.

$$\lim_{h \to 0} \frac{g(a+h) - g(a)}{h}$$

For each of the following, determine whether the expression is positive; negative; or zero. (You may assume each limit exists)

$$\lim_{x \to a} \frac{f(x)}{g(x)}$$

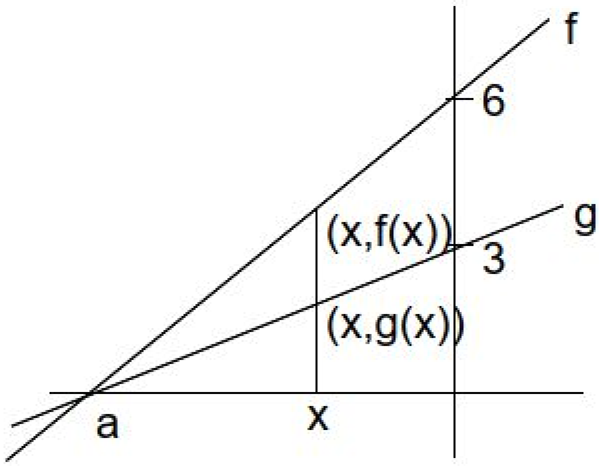
$$\lim_{x \to 0} \frac{f(x)}{g(x)}$$

$$\lim_{x \to c} \frac{f(x)}{g(x)}$$

$$\lim_{h \to 0} \frac{f(c+h) - f(c)}{h}$$

Question L6

The graphs of y = f(x), y = g(x) are shown.



Find $\lim_{x \to a} \frac{f(x)}{g(x)}$.

Then defend your conclusion by providing values of f(x), g(x), and $\frac{f(x)}{g(x)}$ for x near x = a.

Question L7

Sketch a graph of a function y = f(x) with **ALL** of the following properties:

- $\lim_{x \to \infty} f(x) = -1$
- $\lim_{x \to 0} f(x)$ does not exist.
- f(0) = 15.

Question L8

Suppose f(x) has **ALL** of the following properties:

- $\lim_{x \to 3} f(x) = 5$
- $\lim_{x \to \infty} f(x) = -1$
- $\lim_{x \to 0} f(x)$ does not exist.
- f(0) = 15.
- f(x) is never equal to zero.

Explain why f(x) is discontinuous at x = 0.

Is f(x) discontinuous anywhere else? Why/why not?

Question L9

A student tries to find $\lim_{x \to 5} f(x)$. They find the following values:

x	4.9	4.99	4.999	5			
f(x)	105	1015	10015	ERR			

Explain what is wrong with the following statement: "Since f(5) is undefined, $\lim_{x \to 5} f(x)$ does not exist.

Explain why, at this point, it appears that $\lim_{x \to 5} f(x) = \infty$.

The student, being sensible, wants more evidence to support or refute the claim. In the **first** blank column, write down a value of x and f(x) (any value you want) that would *support* the claim that $\lim_{x \to 5} f(x) = \infty$. (You can pick both x and f(x): for example, you might say that x = 10 and f(10) = 25, as long as your proposed values support the claim that $\lim_{x \to 5} f(x) = \infty$)

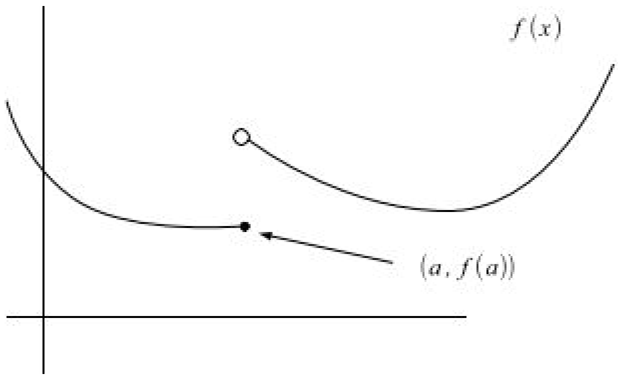
The student, being sensible, wants more evidence. In the **second** blank column, write down a value of x and f(x) (any value you want) that would *refute* the claim $\lim_{x \to 5} f(x) = \infty$.

Explain why, based on the table (including the values you've entered) it would be reasonable to conclude $\lim_{x \to 5^-} f(x) = \infty$.

The student, being sensible, wants more evidence. In the **third** blank column, write down a value of x and f(x) (any value you want) that would *refute* the claim $\lim_{x \to 5^-} f(x) = \infty$.

Question L10

The graph of y= f(x) is shown.



Explain why f(x) is not continuous at x = a. Your explanation **MUST** be based on the definition of continuity.

Explain why f ' (a) does not exist. Your definition **MUST** be based on the definition of the derivative.

Question L11

Let $f(x) = \begin{cases} 5x+5 & \text{if} \quad x < 3 \\ 35-5x & \text{if} \quad x > 3 \\ 19 & \text{if} \quad x = 3 \end{cases}$

Determine whether f(x) is continuous at $x = 3$. If f(x) is not continuous, identify why.

- ○ Not continuous: $\lim_{x \to 3} f(x) \neq f(a)$.
- ○ Not continuous: $f(3)$ is undefined.
- ○ Not continuous: $\lim_{x \to 3} f(x)$ does not exist.
- ○ The function is continuous at $x = 3$.

Question L12

Let $f(x) = \begin{cases} cx + 5 & \text{if } x < 1 \\ 13 - 4x & \text{if } x > 1 \\ d & \text{if } x = 1 \end{cases}$

Find values of c, d that make $f(x)$ continuous at $x = 1$.

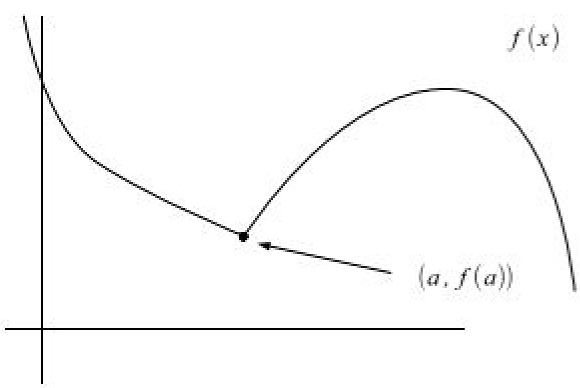
$c =$ ☐ and d = ☐

$\lim_{x \to 1} f(x) =$ ☐

Derivatives

<u>Question D1</u>

The graph of y = f(x) is shown.



What is the **<u>sign</u>** of $\lim_{h \to 0^+} \frac{f(a+h) - f(a)}{h}$? Defend your answer.

What is the **<u>sign</u>** of $\lim_{h \to 0^-} \frac{f(a+h) - f(a)}{h}$? Defend your answer.

What do your results say about the value of f'(a)?

Question D2

Suppose f, f ', g, g ' have the following values.

x	-2	-1	0	1	2
f(x)	9	3	-1	-3	-3
f ' (x)	-7	-5	-3	-1	1
g(x)	49	77	75	49	5
g ' (x)	45	12	-15	-36	-51

Find the derivatives of the following.

h'(2), where h(x) = ln(g(x)).

k'(1), where k(x) = sin (3 f(x) + g(x))

m'(0), where m(x) = sqrt(g(x))

n'(1), where n(x) = f(x) g(x)

p'(3), where p(x) = f(x)/g(x)

q'(0), where q(x) = g[f(x)]

r'(0), where r(x) g(x) + [r(x)]^3 = f(x).

Find the equation of the line tangent to the graph of x^3 + y^3 = f(x) g(x) at x = 3.

Question D3

Suppose $f'(x) = \sqrt{1+x^4}$. Find $\frac{d}{dx}f(x^2)$.

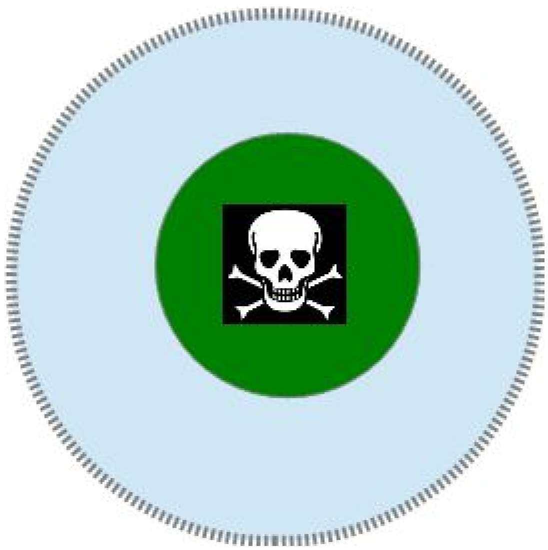
Question

Suppose a tank leaks toxic chemicals. The area affected is a circle centered on the tank. When the spill is first noticed, the circle has a radius of 5000 feet. A response team arrives 30 minutes later, by which time the affected area has grown to a circle with a radius of 5050 feet.

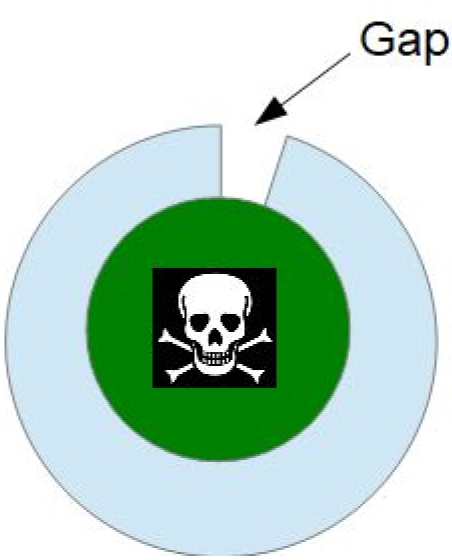
How rapidly was the **radius** of the spill changing between the time the spill was noticed and the arrival of the response team?

How rapidly was the **area** of the spill changing between the time the spill was noticed and the arrival of the response team?

The response team plans to enclose the spill with a circular barrier (see figure). If it will take 90 minutes to set up the circular barrier, how long should it be? Defend your conclusion.



Suppose the response team plans to obtain just enough barrier material to enclose a circular region with a radius of 5200 feet. However, when the material is delivered, they discover that they are 5 feet short; in other words, when they try to enclose a circular area, there is a gap of 5 feet:

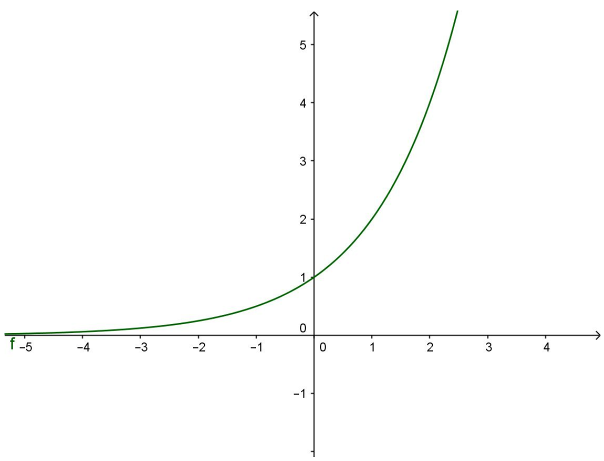


Since the barrier won't work unless it's closed, they have to use a smaller circle. To make the smaller circle, the members of the response team will take sections of the barrier and walk towards the center. About how far towards the center will they have to walk? (a) A very small step towards the center, (b) about a regular walking step towards the center, (c) several giant steps towards the center.

The barrier will enclose a smaller area. How much *less* area will be enclosed? (a) about the area of a smart phone (less than one square foot), (b) about the area of a sheet of paper (about one square foot), (c) about the area of a classroom (about two hundred square feet), (d) about the area of a football field (several thousand square feet).

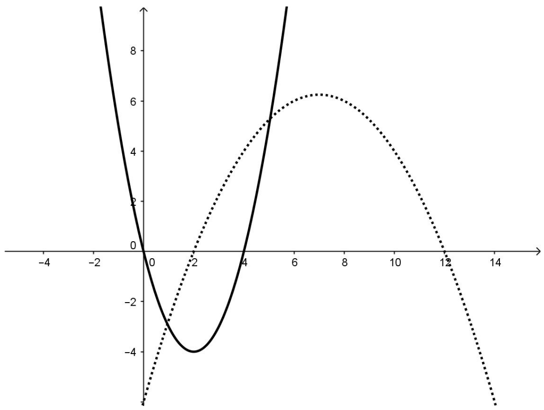
<u>Question D4</u>

A student claims $\frac{d}{dx}2^x = x2^{x-1}$. Use the graph of $y = 2^x$, shown below, to explain why this is not true. **<u>You may not simply state or otherwise refer to the actual derivative of the function</u>**.



<u>Question D5</u>

The graphs of y = f(x) (solid) and y = g(x) (dashed) are shown.



Find the **sign** of the following.

(fg)'(4)

(f/g)'(0)

Question D6

Find the derivative of $f(x)$ everywhere that f is differentiable. Secondly, give the set of points where $f(x)$ is not differentiable.

$$f(x) = \begin{cases} \frac{x}{x+5} & x \leq -3 \\ e^{x+3} - \frac{5}{2} & -3 < x \leq -1 \\ \tan x & -1 < x \leq 0 \\ x^2 & x \geq 0 \end{cases}.$$

Question D7

. Let f and g be differentiable functions on $(-\infty, 2]$. If $f(2) = 1$, $f'(2) = 2$, $g(2) = 3$, and $g'(2) = -1$. Let $h(x)$ be as below. Is $h(x)$ differentiable for all real numbers?

$$h(x) = \begin{cases} f(x)g(x) & x \leq 2 \\ 2x^2 - 3x + 1 & x > 2 \end{cases}.$$

Question D8

Let f(x) be defined as follows.

$$f(x) = \begin{cases} cx^2 & \text{if } x \leq 5 \\ 12 - dx & \text{if } x > 5 \end{cases}$$

Find c, d so that f(x) is differentiable for all real numbers.

Uses of the Derivative

Question U1

Let P(t) model the population of the country, in thousands, *t* years after 2000; let R(t) model the number of doctors in the country, in thousands, *t* years after 2000.

What is the significance of $\frac{P(t)}{R(t)}$?

Can P(t) ever be negative? Why/why not?

Assume P'(t) exists for all t. Can P'(t) ever be negative? Why/why not?

Find an expression for $\frac{d}{dt}\left(\frac{P(t)}{R(t)}\right)$. Assume all required derivatives exist.

Suppose P(t), P'(t), R(t), and R'(t) are always positive. Is it possible for $\frac{d}{dt}\left(\frac{P(t)}{R(t)}\right)$ to be negative? If possible, give a set of positive values P(t), P'(t), R(t), R'(t) that would accomplish this; if not possible, explain why not.

Suppose your goal is to improve medical care in the country (by increasing access to doctors, for example). Do you want $\frac{d}{dt}\left(\frac{P(t)}{R(t)}\right)$ to be positive or negative? Explain why.

Question U2

Some values of y = f(x), f'(x) are given below. Assume y = f(x) and all its derivatives are continuous for all x.

x	0	5	10	15	20
f(x)	-3	8	10	-5	7
f'(x)	-1	-2	3	-2	-10

Which interval(s) are **certain** to contain a solution to f(x) = 0? Defend your conclusion.

Which interval(s) are **certain** to contain a local maximum value of f(x)? Defend your conclusion.

Which interval(s) are **certain** to contain a local minimum value of f(x)? Defend your conclusion.

Which interval(s) are **certain** to contain an inflection point of the graph of y = f(x)? Defend your conclusion.

Question U3

Suppose the only critical values of $f'(x)$, are at $x = 10, 20$, and 30, and $f'(x)$ and $f''(x)$ have the following values:

x	5	10	15	20	25	30	35
$f'(x)$	10	0	-2	0	4	0	-6
$f''(x)$	5	-9	-9	9	10	0	-6

Based on this information,

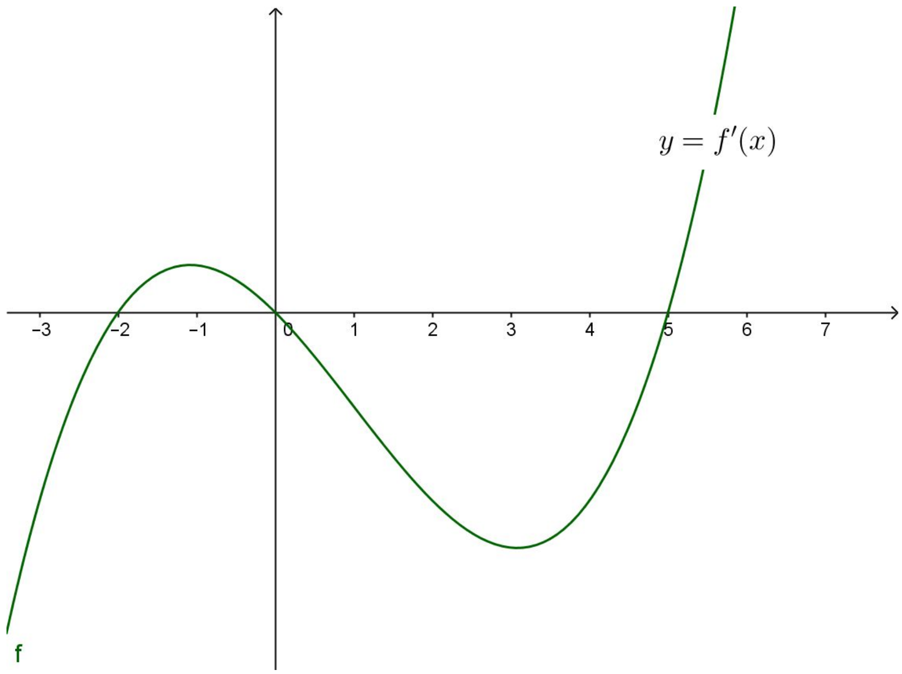
At $x = 10$, you guarantee Select an answer

At $x = 20$, you guarantee Select an answer

At $x = 30$, you guarantee Select an answer

Question U4

The graph of y = f'(x) is shown below. **NOTE**: This is the graph of the *derivative* of f(x). This is **not** the graph of y= f(x).



<u>Reminder</u>: You are given the graph of the derivative.

Find the interval(s) where f(x) is increasing and the interval(s) where it is decreasing.

Find the interval(s) where f'(x) is increasing and the interval(s) where it is decreasing.

The local minimum(s) of f(x) occurs at x =

The local maximum(s) of f(x) occurs at x =

The inflection point(s) of the graph of y = f(x) occur at x =

Suppose you know f(0) = 100. Sketch a graph of y = f(x) over the interval -3 <= x <= 7.

<u>Question U5</u>

Suppose f(x) is increasing and concave down over the interval 5 <= x <= 10.

Select the GREATER of the values. Then defend your answer.

f(7) or f(8)

f ' (7) or f ' (8)

$$\lim_{h\to 0}\frac{f(5+h)-f(5)}{h} \text{ or } \lim_{h\to 0}\frac{f(8+h)-f(8)}{h}$$

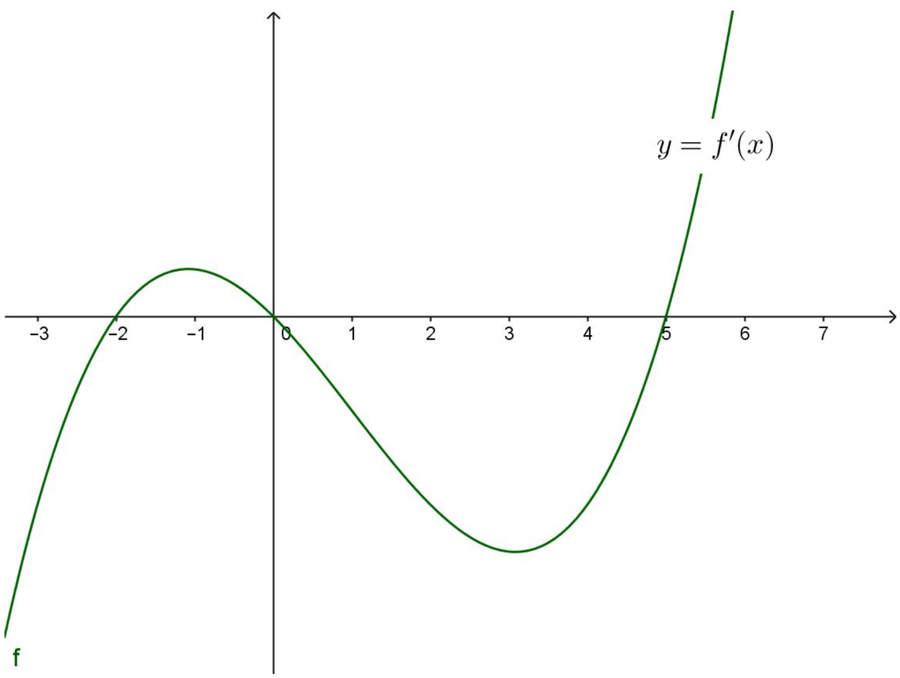
$$\frac{f(8)-f(5)}{8-5} \text{ or } \frac{f(10)-f(8)}{10-8}$$

f ' (8) or 0

f ' ' (8) or 0.

Question U6

The graphs of y = f'(x) is shown over the interval -3 <= x <= 7. **Note that this is the graph of the derivative and NOT the graph of y = f(x)**.



In the interval -3 <= x <= 6, what is the **MAXIMUM** number of times the graph of y = f(x) can cross the x-axis?

Question U7

Sketch a graph of the function with **ALL** of the following properties.

- The graph is continuous for all x.

- The graph of y = f(x) is concave down until x = 3, then concave up after x = 3.
- f'(x) < 0 for x < 3.
- f'(x) > 0 for x > 3.

What can you say about f'(3)? Defend your answer.

Question U8

A tank leaks $0.09\ \frac{\mathrm{m}^3}{\mathrm{hr}}$ of oil into a lake. The oil forms a semicircular disk with a thickness of 10^{-6} meters. How rapidly is the radius of the disk increasing when the radius is 165 meters?

The radius of the disk is increasing by [ ] m/hr (round your answer to 3 decimal places)

Question U9

A particle moves along the curve $y = -9x^2 + 4x + 2$ in such a way that as it moves, $\frac{dx}{dt} = -3$ units/minute.

When the particle reaches the point (5, -203) it is moving [Select an answer] the origin at a rate of [ ] units/minute. (Enter your answer rounded to 3 decimal places)

Question U10

A lighthouse is on an island 400 meters off shore. The lighthouse beam makes a full turn every 14 seconds. How rapidly is the point where the beam meets the shoreline moving *along the shoreline* when the beam meets the shoreline at a point 1500 meters from the lighthouse?

[ ] m/s (enter your answer rounded to 3 decimal places)

Question U11

A coffeeshop determines that it can sell 2000 cups of coffee at a price of \$ 2.00, but for every \$1 increase in the price, their sales will decrease by 2200 cups and for every \$1 decrease, their sales will increase by 2200 cups.

A function giving the revenue earned when the price of coffee is x dollars per cup is $R(x) =$

The coffeeshop can earn a maximum of

when the price of a cup of coffee is \$

(round your answers to two decimal places)

Question U12

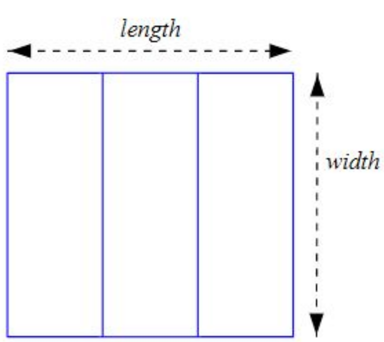
Find the point on the line y = 3x - 7 that is closest to (5, 3).

Question U13

A box with a square base and no top must have a volume of 5000 cubic inches. What dimensions provide the least total area of the box?

Question U14

Brooke wants to enclose a rectangular field with three internal partitions as shown. If there is 15900 feet available for fencing, what dimensions will produce the greatest area? Note that the internal partitions also require fencing.

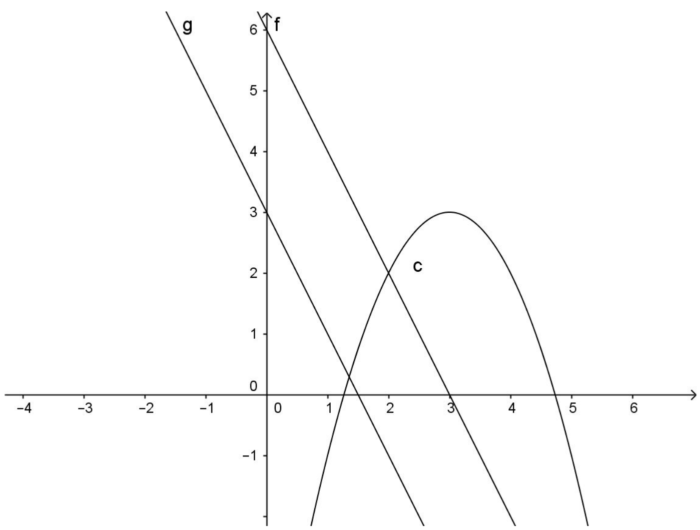


The width should be 1987.5 feet and the length should be 3975 feet. (Round your answers to two decimal places)

Antiderivatives

Question A1
An experiment begins at t = 0, when an object with a velocity of 12 m/s enters a tube. While inside the tube, the object's velocity increases by 8 + 3t m/s^2, where t is the number of seconds after the experiment begins. Find s(t), the object's distance from the entrance to the tube t seconds after the experiment begins.

Question A2
A student draws the graphs of y = f(x), y = f'(x), and y = g(x). Unfortunately, they didn't label the graphs, and they mixed up the order. Rather than giving them a "0" for the assignment, their professor attempts to determine which graph is which. What graph corresponds to which function or derivative?



Question A3
Suppose f(x) is differentiable, where f(0) = 3, f'(0) = 4, and f''(x) < 0 for all x. Which of these could be true?

(a) f(2) is undefined
(b) f(2) = 11
(c) f(2) = 8

(d) $\lim_{h \to 0} \dfrac{f(2+h) - f(h)}{h} = 5$

(e) $\lim_{h \to 0} \dfrac{f(2+h) - f(h)}{h} = 2$

Integrals

<u>Question R1</u>

Let

$$f(x) = \begin{cases} 3\cos(2x) & x \leq 0 \\ 2x^2 - 5 & 0 < x \leq 4 \\ 7 & 4 < x \end{cases} .$$

Compute

$$\int_{\pi}^{10} f(x)\, dx.$$

<u>Question R2</u>

Suppose you know F(5) = 10, F(3) = 2, where F'(x) = f(x). Find the following.

$$\int_3^5 f(x)\ dx$$

$$\int_3^5 (x^2 + 5f(x) - 8)\ dx$$

$$\int_2^2 f(x)\ dx$$

$$\int_3^5 \frac{f(x)}{F(x)}\ dx$$

$$\int_3^5 f(x) \cos F(x)\ dx$$

<u>Question R3</u>

Suppose f(x) satisfies f'(x) >0 for all x. Further, suppose f(x) has the following values:

x	0	1	2	3
f(x)	4	5	8	10

- Find an approximate value for $\int_0^3 f(x)\ dx$.
- Is your approximate value greater or less than the actual value of $\int_0^3 f(x)\ dx$? Defend your conclusion.

<u>Question R4</u>

Suppose f(x) is positive, continuous, and f '(x) <= 0 for all x in 0 <= x <= 10, and $\int_0^{10} f(x)\ dx = 100$, $\int_8^{10} f(x)\ dx = 10$.

Find $\int_0^8 f(x)\ dx$.

Find the least and greatest possible values of f(8).

<u>Question R5</u>

Sketch a graph of y = f(x) that satisfies the following requirements on the interval 0 <= x <= 10.

- f(x) is positive for all x in 0 <= x <= 10
- f(x) is <u>**NOT**</u> continuous at x = 5.
- $f'(x)$> 0 for all x not equal to 5.

- $f''(x)$< 0 for 0<= x < 5, and $f''(x)$>0 for 5 < x <= 10.
- $\int_0^{10} f(x)\ dx = 75$

Question R6

Suppose v(t) is the velocity, in m/s, of an object t seconds after an experiment begins. Which of the following can be used to find *acceleration* of the object?

(a) $\int v(t)\ dt$
(b) v(t)
(c) v'(t)
(d) v''(t)

Which of the following can be used to find the distance traveled by the object?

(a) $\int v(t)\ dt$
(b) v(t)
(c) v'(t)
(d) v''(t)

Question R7

Suppose f(x) is positive, continuous, increasing, and concave up over the interval 0 <= x <= 3. For each pair of values, select the greater value. Defend your conclusion.

The left approximation or the right approximation.

The left approximation using n = 500 or the left approximation using n = 1000.

The right approximation or $\int_0^3 f(x)\ dx$

$\int_0^1 f(x)\ dx$ or $\int_1^2 f(x)\ dx$

$3\int_0^1 f(x)\ dx$ or $\int_0^3 f(x)\ dx$

Question R8

Suppose $F(x) = \int_0^x \sin t^2\ dt$, and $G(x) = \frac{F(x)}{x^2+1}$ **DO NOT** attempt to evaluate the integral: you will only waste your time.

Find the following.

F(0)

F'(0)

G'(0)

Suppose H(x) = F(x^2). Find H'(x).

Question R9

The graph of $y = f(x)$ is given below. Evaluate $\int_0^4 f(x)dx$.