This is a collection of questions that will be representative of those on the final exam. This is **NOT** a sample final exam: the final exam may have more or fewer questions, or the questions may be asked in a different format.

There **WILL** be other types of questions on the final exam. However, about 80% of your grade on the exam will be based on questions similar to the following. (The remainder will be based on everything else you did in the course)

Integration Techniques

Question T1

Suppose f(x) has the following properties:

- f(5) = 25 and f(0) = 8
- $\int_0^5 f(x)\ dx = 3$
- $\int_0^5 xf(x)\ dx = 10$

Evaluate: $\int_0^5 (x^2 + 3x + 7)f'(x)\ dx$

Question T2

Prepare (but do not evaluate) the following integral using integration by parts.

$$\int \frac{1}{(x+3)(9x^2+4)}\ dx$$

Question T3

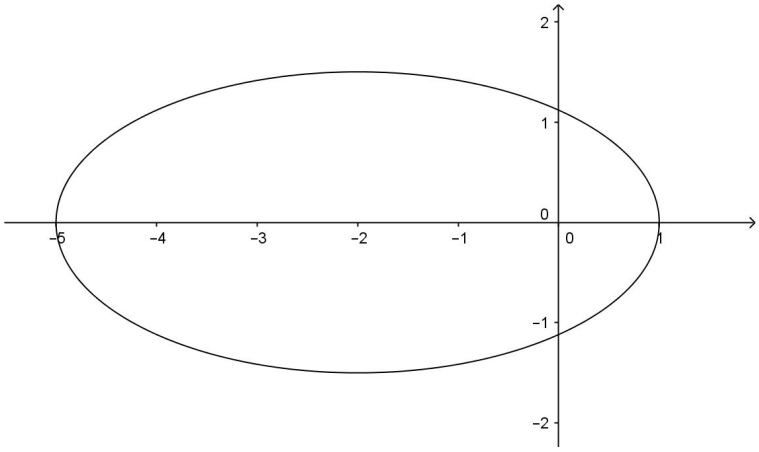
Rewrite (but do not evaluate) the following integral using a trigonometric substitution:

$$\int \frac{1}{(x+3)(9x^2+4)}\ dx$$

Applications of the Integral

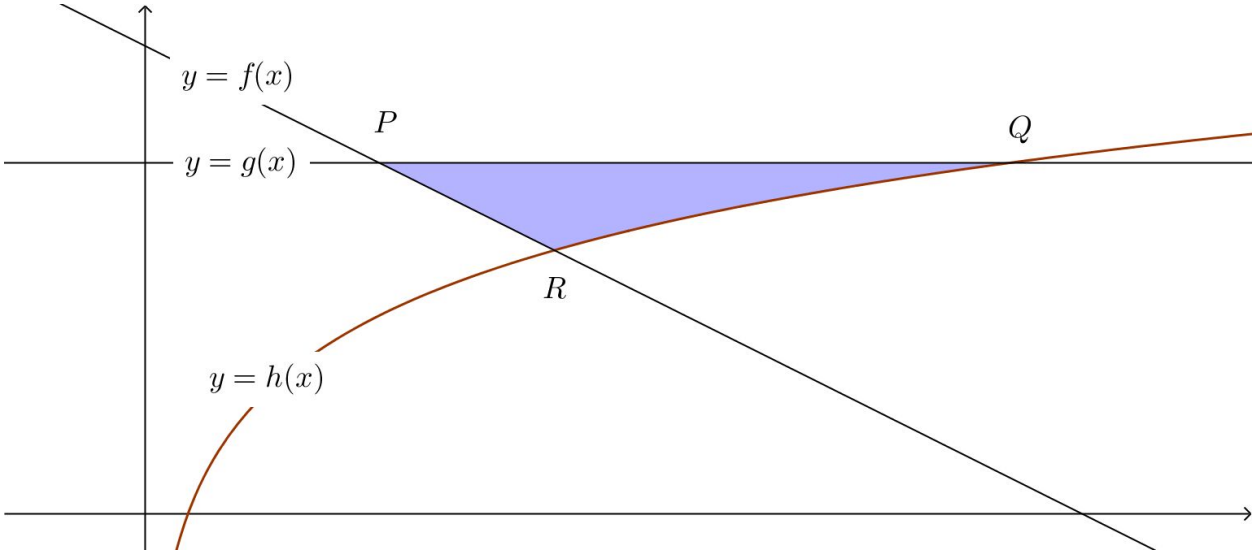
Question A1

An asteroid moves in an elliptical orbit with equation $x^2 + 12x + 4y^2 + 20y = 400$ (the graph is shown below). Set up but **do not evaluate** an expression to determine the distance the asteroid travels in one orbit. (Assume the asteroid is moving clockwise along the ellipse)



Question A3

Consider the region PQR shown in the figure.



Let P = (1, 4), R = (2, 2), and Q = (8, 4).

Suppose this region is to be revolved around the x-axis. to produce a solid of revolution.

Set up two ways, don't integrate.

You have learned three different methods of using integration to find volumes of solids of revolution: shells, disks, and washers.

Which of these methods is **not** suitable for computing the volume of the solid of revolution? Explain why.

Write down integral expressions corresponding to computing the volumes using **each** of the other two methods.

Suppose f(x) and g(x) are linear functions, but h(x) = ln(ax + b) for some values a, b (**DO NOT** attempt to find a, b: their values are irrelevant for this question). Which of your two integral expressions would be easier to evaluate? Why?

L'Hopital's Rule

Question H1

Suppose $f(x)$ and $g(x)$ have the following values:

- $f(10) = 1$ and $f'(10) = -6$,
- $g(10) = 1$ and $g'(10) = 6$.

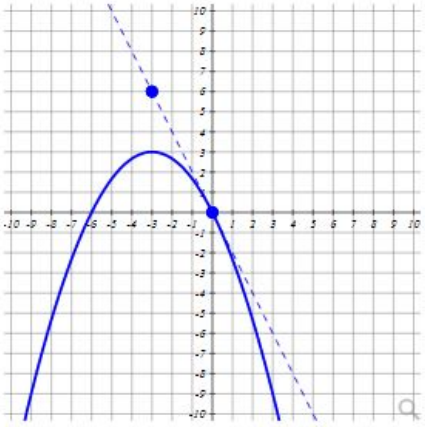
Assume all functions are continuous. Find the following.

$$\lim_{x \to 10} \frac{[f(x)]^2 - 1}{\ln g(x)} =$$

$$\lim_{x \to 10} \frac{(f(x) - 1)^2}{(g(x) - 1)^2} =$$

Question H2

The graph of $y = f(x)$ (solid) and its tangent line at $x = 0$ (dashed) is shown; a second point on the tangent line is also marked. Assume $f'(x)$ is continuous.



Find

$\lim_{x \to 0} \frac{f(x)}{x} =$ [ ]

$\lim_{x \to 0} \frac{f(x)}{3e^x - 3} =$ [ ]

Question H3

Suppose $f(8) = 6$ and $f'(8) = 4$.

Find the following.

$$\lim_{x \to 8} \frac{e^{f(x)} - e^6}{x^2 - 4x - 32} = \text{[ ]}$$

$$\lim_{x \to 8} \frac{e^{f(x)-6} - f(x) + 5}{x^2 + 16x + 64} = \text{[ ]}$$

Improper Integrals

Question M2

Assume f(x) is continuous and positive everywhere, and that $\int_1^\infty f(x)\ dx$ converges. Find the following.

a) Assuming it exists, $\lim_{x\to\infty} f(x)$

b) Assuming it exists, $\lim_{x\to\infty} f'(x)$

Decide on the convergence or divergence of the following ("cannot be determined" is also an option). Defend your conclusion.

$$\int_{10}^\infty f(x)\ dx$$

$$\int_1^\infty f(x+3)\ dx$$

$$\int_1^\infty (f(x)+3x)\ dx$$

$$\int_1^\infty \frac{f(x)}{x}\ dx$$

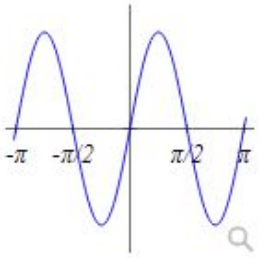
$$\int_1^\infty f(x^2)\ dx$$

$$\int_1^\infty f(x)\sin x\ dx$$

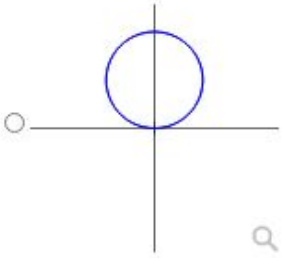
Polar Coordinates

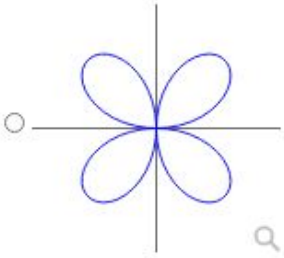
Question R1

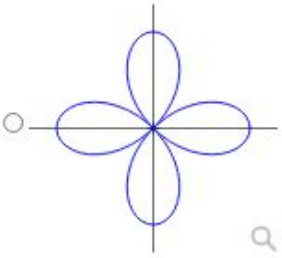
Jamie tried to produce the polar graph $r = f(\theta)$, but instead produced the rectangular graph of $y = f(x)$



Which of the following is the graph she should have produced?

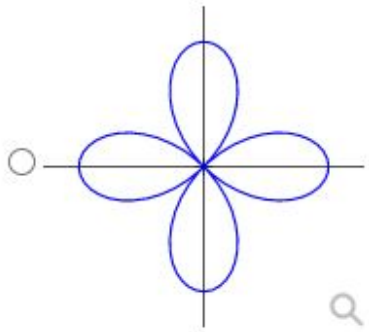


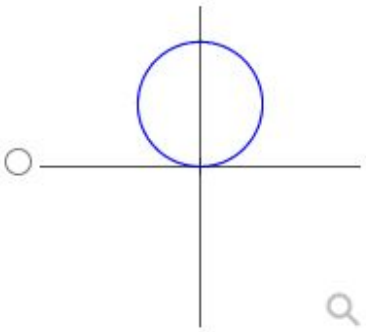


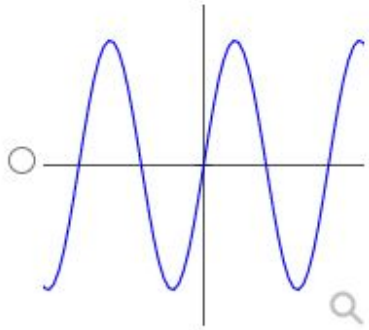


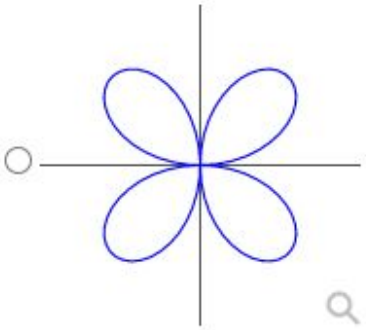
Question R3

Suppoes $a > 0$. Which of the following might represent the polar graph $r = a \sin 2\theta$?



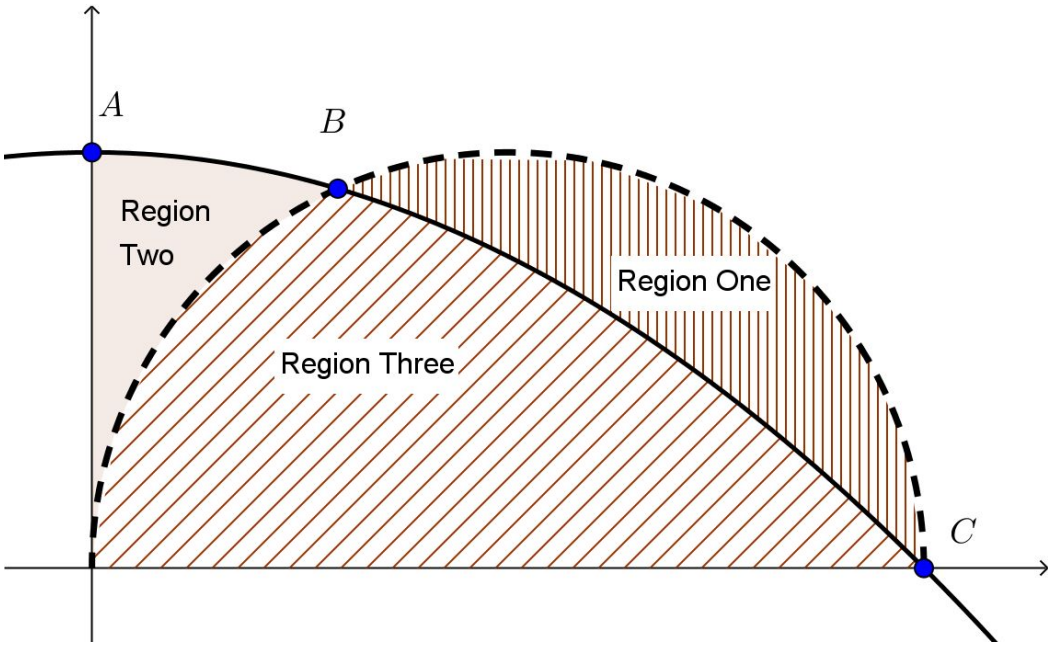






Question R2

The graph shows $r = f(\theta)$ (solid) and $r = g(\theta)$ (dashed); the points have coordinates A = [4,pi/2], B = [3, pi/3], C = [4, 0]. Write an expression for the area of Region One; Region Two; and Region Three.



Series Convergence

Question S1

During the semester, you have learned or utilized the following tests for series convergence: nth term test; ratio test; root test; limit comparison test; integral test.

Consider the series $\sum \frac{(2n+5)^3}{\sqrt{n^8+5n+23}}$.

Of the five series convergence tests, identify two tests which will be inconclusive and will not provide information about whether the series converges or diverges; show your work.

Identify one test which will be conclusive; use it to determine whether the series converges or diverges.

Predict the result of the remaining test (in other words, will the test be inconclusive; predict the series converges; or predict the series diverges).

Power Series

Question P1

Suppose f(x) has the following properties:

- f(x) and all its derivatives exist at x = 5.
- f(5) = 8.
- $f'(x) = \frac{1}{[f(x)]^2}$ for all x.

Question P3

Suppose $f(x) = \sum_{n=0}^{\infty} \left(\frac{3}{5}\right)^n (2x)^n$.

- Find a series expression for $\int x^3 f(x)\ dx$.
- If possible, use your series to approximate $\int_0^2 x^3 f(x)\ dx$. If not possible, explain why not.

Question P4

Suppose

$$f(x) = \sum_{n=0}^{\infty} \frac{2^n}{n!}(x-7)^n$$

To determine $f(6.9)$ to within 0.0001, it will be necessary to add the first [ ] terms of the series.

$f(6.9) \approx$ [ ] (Enter the answer accurate to four decimal places)